130. *5-dimensional Orientable Submanifolds of R^7. I*

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Introduction. It is well known that the odd dimensional number space R^{2n+1}, the odd dimensional sphere S^{2n+1} and orientable hypersurfaces of an almost complex manifold etc. admit an almost contact structure.

The main purpose of this paper is to show that, using the vector cross product induced by Cayley numbers, any 5-dimensional orientable submanifold of any 7-dimensional parallelizable manifold admits an almost contact structure.

2. Basic informations.

(a) Almost contact manifolds.

An almost contact structure (ϕ, ξ, η) on a $(2n+1)$-dimensional C^∞ manifold M is given by a tensor field ϕ of type (1, 1), a vector field ξ and a 1-form η called the *contact form* such that

(1) $$\eta(\xi)=1,$$

(2) $$\phi(\xi)=0, \quad \eta\circ\phi=0,$$

(3) $$\phi^2=-I+\eta(\cdot)\xi,$$

where I is the identity transformation field.

If M has a (ϕ, ξ, η)-structure then we can find a Riemannian metric $\langle\ ,\ \rangle$ such that

(4) $$\eta=\langle\xi, \cdot\rangle,$$

(5) $$\langle\phi X, \phi Y\rangle=\langle X, Y\rangle-\eta(X)\eta(Y),$$

for any vector fields X, Y on M, so that ϕ is skew symmetric with respect to $\langle\ ,\ \rangle$. M is then said to have a $(\phi, \xi, \eta, \langle\ ,\ \rangle)$-structure and $\langle\ ,\ \rangle$ is called the *associated Riemannian metric* of (ϕ, ξ, η).

(b) Vector cross products on certain 7-dimensional Riemannian manifolds.

The *vector cross product* on a certain 7-dimensional Riemannian manifold $\bar{M}$ is a linear map $P: V(\bar{M})\times V(\bar{M})\to V(\bar{M})$ (writing here $P(\bar{X}, \bar{Y})=\bar{X}\otimes\bar{Y}$) satisfying the following conditions:

(6) $$\bar{X}\otimes\bar{Y}=-\bar{Y}\otimes\bar{X},$$

(7) $$\langle\bar{X}\otimes\bar{Y}, \bar{Z}\rangle=\langle\bar{X}, \bar{Y}\otimes\bar{Z}\rangle,$$

(8) $$(\bar{X}\otimes\bar{Y})\otimes\bar{Z}+\bar{X}\otimes(\bar{Y}\otimes\bar{Z})=2\langle\bar{X}, \bar{Z}\rangle\bar{Y}-\langle\bar{Y}, \bar{Z}\rangle\bar{X}-\langle\bar{X}, \bar{Y}\rangle\bar{Z},$$

where $V(\bar{M})$ denotes the ring of differentiable vector fields on $\bar{M}$ and $\bar{X}, \bar{Y}, \bar{Z}\in V(\bar{M})$. Any parallelizable 7-dimensional Riemannian mani-