120. *On a Subclass of M-Spaces*

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1. Introduction. In the present paper all spaces are Hausdorff. In a previous paper [3], K. Morita defined M-space, which is an important generalization of metric and compact spaces. A space X is an M-space iff there is a normal sequence $\{\mathcal{U}_i : i=1, 2. \cdots\}$ of open covers of X satisfying condition (M_0) below;

$$(M_0)\quad \begin{cases} \text{If } \{x_i\} \text{ is a sequence of points in } X \text{ such that} \\ x_i \in \mathrm{St}(x_0, \mathcal{U}_i) \text{ for all } i \text{ and for fixed } x_0 \text{ in } X, \\ \text{then } \{x_i\} \text{ has a cluster point.} \end{cases}$$

Unfortunately, the product of M-spaces may not be M, for which reason T. Ishii, M. Tsuda and S. Kunugi [2] have defined a class $\mathfrak{C}$ of spaces. A space X is of class $\mathfrak{C}$ iff there is a normal sequence $\{\mathcal{U}_i : i=1, 2, \cdots\}$ of open covers of X satisfying condition $(*)$ below:

$$(*)\quad \begin{cases} \text{If } \{x_i\} \text{ is a sequence of points of } X \text{ such that} \\ x_i \in \mathrm{St}(x_0, \mathcal{U}_i) \text{ for all } i \text{ and for fixed } x_0 \text{ in } X, \\ \text{then there is a subsequence } \{x_{i(n)}\} \text{ which has} \\ \text{compact closure.} \end{cases}$$

Ishii, Tsuda and Kunugi have proved in [2] that if a space X is of class $\mathfrak{C}$, then $X \times Y$ is M for any M-space Y; and that the product of countably many spaces of class $\mathfrak{C}$ is also of class $\mathfrak{C}$. They also prove that among the M-spaces belonging to class $\mathfrak{C}$ are:

(a) first countable spaces,
(b) locally compact spaces,
(c) paracompact spaces.

The purpose of this paper is to introduce weakly-k spaces (which contain (a) and (b) above) and weakly para-k spaces (which contain (a), (b), and (c) above), in order to improve Ishii, Tsuda and Kunugi's result as follows:

Theorem 1.1. *Given a space X, the following are equivalent*:
(a) *X is of class $\mathfrak{C}$.*
(b) *X is M and weakly-k.*
(c) *X is M and weakly para-k.*

The spaces are defined as follows:

Definition 1.2. X is *weakly-k* iff: given $F \subseteq X$, $F \cap C$ is finite for all C compact in X implies F closed.