119. *A Note on M-Space and Topologically Complete Space*

By Jun-iti NAGATA
Department of Mathematics, University of Pittsburgh

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In the previous paper [4] we have proved that every paracompact M-space with weight $|A|$ (=the cardinality of the set A) is the perfect image of a closed subset of $D(A)$ and a subset of $N(A)$, where $D(A)$ is Cantor discontinuum (=the product of two points discrete spaces D_α, $\alpha \in A$), and $N(A)$ is Baire's 0-dimensional space (=the product of countably many copies of the discrete space A), and also stated the following theorem without proof. (Throughout this paper we assume that A is an infinite set and that spaces are Hausdorff. As for terminologies and symbols in the present paper, see J. Nagata [3] and [4].)

Theorem 1. *A space X with weight $|A|$ is a paracompact M-space iff (=if and only if) it is homeomorphic to a closed subset of $S\times P(A)$, where S is a subspace of generalized Hilbert space $H(A)$, and $P(A)$ is the product of the copies I_α, $\alpha\in A$ of the unit interval* [0, 1].

The purpose of the present paper is to give a proof of Theorem 1 and extend our study to paracompact, topologically complete spaces (in the sense of E. Čech), which form an important subclass of paracompact M-spaces.

Proof of Theorem 1. Since the sufficiency of the condition is obvious, we shall prove only the necessity. There is a perfect map (=mapping) from X onto a metric space Y with weight $\leqq |A|$. Let $\{f_\lambda \mid \lambda\in A\}$ be a collection of continuous functions $|X\to[0, 1]$ such that for each point x of X and each nbd (=neighborhood) M of x, there is $\lambda\in A$ for which $f_\lambda(x)=1$, $f_\lambda(X-M)=0$.
Then we define a map $h\mid X\to Y\times P(A)$ by

$$h(X)=\varphi(x)\times(f_\lambda(x)\mid \lambda\in A),\ x\in X.$$

It is obvious that h is one-to-one and continuous. It is also easy to show that h^{-1} is continuous. Hence h is a topological map. To show that $h(X)$ is closed in $Y\times P(A)$, let $z=y\times(q_\lambda\mid\lambda\in A)\in Y\times P(A)-h(X)$. Then $\varphi^{-1}(y)\cap[\bigcap_{\lambda\in A} f_\lambda^{-1}(q_\lambda)]=\phi$, because otherwise for every point x in the non-empty intersection $h(x)=z$ holds, and thus $z\in h(X)$. Since each $f_\lambda^{-1}(q_\lambda)$ can be expressed as $f_\lambda^{-1}(q_\lambda)=\bigcap_{n=1}^{\infty} f_\lambda^{-1}\left(\left[q_\lambda-\frac{1}{n}, q_\lambda+\frac{1}{n}\right]\right)$ ([ ] denotes a closed interval.) and since $\varphi^{-1}(y)$ is compact, there are $\lambda_1, \cdots, \lambda_k\in A$ and a