116. *Stability Problems on Difference and Functional-differential Equations*

By Shohei SUGIYAMA
Department of Mathematics, School of Science and Engineering, Waseda University, Tokyo

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In this paper, we shall show some theorems on the stability problems of difference and functional-differential equations by means of two methods, one of which is directly dependent on the forms of equations and the other is to make use of Lyapunov functionals.

1. Definition of stability. Before stating the definitions of stability, it is convenient to introduce two norms and a family of functions. Let $f(t)$ be a function with ith component $f_i(t)$ $(i=1, \cdots, n)$ defined for an interval I. Then we define two norms such that $|f(t)|=\max_{1\leqq i\leqq n}|f_i(t)|$ for any $t\in I$ and $\|f\|=\sup_{t\in I}|f(t)|$. Let $\mathcal{S}$ be a family of functions which have the following properties:

(i) every function in $\mathcal{S}$ is defined for $s\in[-1, 0)$;

(ii) every function in $\mathcal{S}$ has a limit as $s\to -0$.

Now we shall consider a difference equation[1)]

$$x(t)=f(t, x(t-1)), \tag{1}$$

where $f(t, x)$ is defined for $t_0+k\leqq t<t_0+k+1$ $(k=0, 1, \cdots)$ and $|x|<H$, for any fixed x the limit as $t\to t_0+k-0$ $(k=1, 2, \cdots)$ exists, and $f(t, 0)=0$ for any fixed t. Then we suppose that the difference equation (1) has a solution $x(t)$ for $t\geqq t_0$ such that $|x(t)|<H$ under the initial condition

$$x(t)=\varphi(t), \quad t_0-1\leqq t<t_0. \tag{2}$$

Here the initial function $\varphi(t)$ is a given function defined for $t\in[t_0-1, t_0)$, has a limit as $t\to t_0-0$, and satisfies $\|\varphi(t_0+s)\|<H$, where the norm is defined as before, if we consider the function $\varphi(t_0+s)$ to be in $\mathcal{S}$ as s varies over the interval $[-1, 0)$. If we denote by $x(t, t_0, \varphi)$ the solution of (1) with the initial condition (2), the stability of the trivial solution of (1) will be defined following those of functional-differential equations.

Definition 1. The trivial solution of (1) is said to be *stable* if for any given $\varepsilon>0$ there exists a $\delta(\varepsilon, t_0)$ such that $\|\varphi(t_0+s)\|<\delta(\varepsilon, t_0)$ implies $|x(t, t_0, \varphi)|<\varepsilon$ for any $t\geqq t_0$.

1) In this paper, every equation will be treated in the n-dimensional vector space.