115. *On the Schur Index of a Monomial Representation*

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In this note we give a method of determing the Schur index of a monomial representation of a finite group which is induced from a linear character of its normal subgroup. At the same time we obtain some other results which are useful in the theory of Schur index.

Notation and Terminology. G denotes a finite group whose unit element is 1. $|G|$ is the order of $G\cdot K$ is any given field of characteristic 0 and Ω the algebraic closure of K. An irreducible character χ of G always means an absolute one afforded by a representation of the group algebra ΩG over Ω. $m_K(\chi)$ is the Schur index of χ over K. $K(\chi)$ is the field obtained from K by adjunction of all values $\chi(g)$, $g\in G$. $\mathfrak{G}(K(\chi)/K)$ is the Galois group of $K(\chi)$ over K. For $\tau\in\mathfrak{G}(K(\chi)/K)$, χ^τ is the character of G defined by $\chi^\tau(g)=\chi(g)^\tau$. $e(\chi)=|G|^{-1}\chi(1)\sum_{g\in G}\chi(g^{-1})g$ is the minimal central idempotent of ΩG corresponding to χ. $a(\chi)=\sum_{\tau\in\mathfrak{G}(K(\chi)/K)}e(\chi^\tau)$ is the identity of the simple component A of KG with the property $\chi(A)\neq 0$ [2, V, 14. 12]. If H is a subgroup of G and ψ a character of H, ψ^G denotes the character of G induced from ψ. For a ring R and an integer n, R_n is the total matric algebra of degree n over R.

Lemma. *Let H be a subgroup of G and $Hg_1,\cdots,Hg_n$ all the distinct right cosets of H in G. Let ψ be an irreducible character of H such that ψ^G is irreducible. For simplicity, set $e_i=g_i^{-1}e(\psi)g_i$ $(i=1,\cdots\cdots,n)$. Then we have* (i) $e(\psi^G)=\sum_{i=1}^n e_i$, (ii) $e(\psi^G)\Omega G=e_1\Omega G+\cdots+e_n\Omega G$, (iii) $e_ie_j=0$ $(i\neq j)$, $e_ie_i=e_i$, $1\leq i,\ j\leq n$, (iv) $(\psi^\tau)^G=(\psi^G)^\tau$ *for any* $\tau\in\mathfrak{G}(K(\psi)/K)$.

Proof. (i) $e(\psi^G)=|G|^{-1}\psi^G(1)\sum_{g\in G}\psi^G(g^{-1})g=|H|^{-1}\psi(1)\sum_{g\in G}$

$$\sum_{i=1}^n\psi(g_ig^{-1}g_i^{-1})g=\sum_{i=1}^n g_i^{-1}\{|H|^{-1}\psi(1)\sum_{h\in H}\psi(h^{-1})h\}g_i=\sum_{i=1}^n e_i,$$

where $\psi(g)=0$ for $g\notin H$. (ii) It can be easily seen that $e(\psi)\Omega G\simeq e_i\Omega G$ $(i=1,\cdots,n)$ as right ΩG-modules and that $\dim_\Omega e(\psi)\Omega G=n\ \psi(1)^2$ and that $e(\psi^G)\Omega G\subset e_1\Omega G+\cdots+e_n\Omega G$. Hence, $(n\ \psi(1))^2=\dim_\Omega e(\psi^G)\Omega G\leq\dim_\Omega\{e_1\Omega G+\cdots+e_n\Omega G\}\leq n^2\psi(1)^2$. This proves (ii). (iii) We observe that $e_i=e(\psi^G)e_i=e_1e_i+\cdots+e_ie_i+\cdots+e_ne_i$. Since $e_1\Omega G+\cdots+e_n\Omega G$ is a direct sum, it follows that $e_ie_j=0$ $(i\neq j)$, $e_ie_i=e_i$.