113. On the Projective Cover of a Factor Module Modulo a Maximal Submodule[*]

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1. Let R be a ring with 1 which has the Jacobson radical $J(R)$. In [3], Koh has proved the following:

Every irreducible right R-module has a projective cover if and only if R is semiprimary and for any nonzero idempotent $x+J(R)$ in $R/J(R)$ there exists a nonzero idempotent e in R such that $ex-e \in J(R)$.

The purpose of the present paper is, as a generalization of the result of Koh, to show the following theorem:

Theorem. *Let $M=M_R$ be a projective co-atomic module. Then the following statements are equivalent*:

(1) *For every maximal submodule I of M, M/I has a projective cover.*

(2) *$M/J(M)$ is semisimple and for any nonzero idempotent $\hat{s} \in \hat{S}$ there exists nonzero idempotent $e \in S$ such that $\hat{e}\hat{s}=\hat{e}$.*

2. Let $M=M_R$ be an unital right R-module. We write $J(M)$ for the radical of M and $\bar{M}$ for the factor module $M/J(M)$. Let $S=\mathrm{Hom}_R(M, M)$ and let $\hat{S}=\mathrm{Hom}_R(\bar{M}, \bar{M})$. As usual, we write these endomorphisms on the left of their arguments. We note that every $s \in S$ induces an $\hat{s} \in \hat{S}$, since $sJ(M) \subseteq J(M)$. For any submodule U of M, we denote by ν_U the natural epimorphism $M \to M/U$.

A submodule A of M is called *small* if $A+B=M$ for any submodule B of M implies $B=M$. A *projective cover* of M is an epimorphism of a projective module P onto M with small kernel.

We call M is *co-atomic* if every proper submodule of M is contained in a maximal submodule of M. As is easily seen, if M is co-atomic, then $J(M)$ is small in M (cf. [5]). It is well known that M has a maximal submodule if M is projective (cf. [1]), and we can show that semi-perfect modules defined in [4] are co-atomic as follows: Let T be any proper submodule of a semi-perfect module M, and let $P \to M/T \to 0$ be a projective cover of M/T with kernel K. Then $P/K \cong M/T$ and, since any maximal submodule of P contains K, T is contained in a maximal submodule of M as desired.

Lemma 1. *Let M be a projective module and I a maximal*

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