15. *Remark on the $A^p(G)$-algebras*[*)]

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1. Introduction. Let G denote a locally compact abelian topological group with character group $\hat{G}$, and dx (respect $d\hat{x}$) expresses the integration over G (resp. $\hat{G}$) with respect to the Haar measure. For $1 \leq p < \infty$, $A^p(G)$ denotes the linear space of all complex-valued functions in $L^1(G)$ whose Fourier transforms are in $L^p(\hat{G})$. As the linear space $A^p(G)$ is normed by $\|f\|^p = \|f\|_1 + \|\hat{f}\|_p$, then $A^p(G)$ is a semi-simple commutative Banach algebra under convolution as multiplication (see Larsen, Liu and Wang [2]). In this note, we shall show that it is regular and that some local properties hold in it (cf. Rudin [5], section 2.6). It is also proved that the abstract Silov's theorem (see Loomis [4] p. 86) holds for $A^p(G)$. The standard proof of this theorem in $L^1(G)$ (cf. Loomis [4] p. 151) seems to depend upon the uniform boundedness of the approximate identity. The author proved that the approximate identity exists for $A^p(G)$ but uniformly bounded in general (see Lai [3]). However a similar proof is obtained despite of the fact that the approximate identity in $A^p(G)$ is unbounded.

2. Closed ideals and locally properties in the algebra $A^p(G)$.

Since $A^p(G)$ has an approximate identity in the sense of Theorem 1 in Lai [3], the following proposition is immediately.

Proposition 1. *The set J of all functions of $A^p(G)$ such that the Fourier transforms have compact supports in $\hat{G}$ is a dense ideal in $A^p(G)$ with respect to A^p-topology.*

The following theorem proved for $L^1(G)$ in Loomis [4: Theorem 31 F]

Theorem 2. *A closed subset I of $A^p(G)$ is an ideal if and only if it is a translation invariant subspace.*

Proof. The necessity is immediate since $A^p(G)$ has approximate identity and the translation operator is a multiplier.

For the sufficiency, we suppose that I is a closed translation invariant subspace and consider the mapping $f \to (f, \hat{f})$ of $A^p(G)$ in $L^1(G) \times L^p(G)$, so that each continuous linear functional of $A^p(G)$ may

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