13. *On Vector Measures. I*

By Sachio OHBA

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1. Introduction. In [1] Dinculeanu and Kluvanek have proved the following result.

Let S be a set, R a tribe (σ-ring) of subsets of S, X a locally convex linear space with topology defined by a family $\{\| \ \|_p\}_{p\in P}$ of semi-norms, and $m; R\to X$ a vector measure. Then for every $p\in P$ there exists a finite non-negative measure ν_p on R such that

(1) $\lim\limits_{\nu_p(A)\to 0} \|m(A)\|_p=0$

(2) $\nu_p(E)\leqq \sup\{\|m(A)\|_p ; A\subset E, A\in R\}$ ([1] Theorem 1)

They also raised the following problem: whether this theorem remains valid if the tribe is replaced by a semi-tribe (δ-ring)? In this paper we shall give the negative answer for the problem. And in case R is a semi-tribe we shall show that the above theorem remains true under a weaker property than (1) and property (2). (cf. Theorem 1)

In this paper we suppose that X is a normed space in order to simplify the proof.

2. Vector measures. Definition 1. Let S be a set. A nonvoid class R of subsets of S is called a semi-tribe (δ-ring) if;

(1) $A, B\in R \Rightarrow A\cup B\in R,\ A-B\in R.$

(2) $A_n\in R\ (n=1,2,\cdots)\Rightarrow \bigcap\limits_{n=1}^{\infty} A_n\in R.$

From this definition it follows that a semi-tribe R has the following properties.

(3) $A_n\in R,\ A\in R$ and $A_n\subset A (n=1,2,\cdots)\Rightarrow \bigcup\limits_{n=1}^{\infty} A_n\in R$

(4) if we set $R_A=\{B\cap A ; B\in R\}$ for any $A\in R$, then R_A is a tribe on A.

Suppose that X is a normed space and $\tilde{X}$ its completion.

Definition 2. Let R be a clan (ring). A set function m defined on R with values in X is called a vector measure if the following conditions are satisfied

(1) $m(\emptyset)=0$

(2) for every sequence $\{E_n\}$ of mutually disjoint sets of R such that $E=\bigcup\limits_{n=1}^{\infty} E_n\in R,\ m(E)=\sum\limits_{n=1}^{\infty} m(E_n).$

For every $E\in R$, we set $\tilde{m}(E)=\sup\{\|m(A)\| ; A\subset E, A\in R\}$. Then it is easy to see that $\tilde{m}$ is increase, subadditive on R.