5. *wM*-Spaces and Closed Maps

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1. Introduction. In our previous paper [5], we introduced the notion of wM-spaces, which is a generalization of M-spaces (due to K. Morita [8]). A topological space X is called a wM-space if there exists a sequence $\{\mathfrak{A}_n\}$ of open coverings of X satisfying the condition below:

$$(\mathrm{M}_2)\begin{cases}\text{If } \{K_n\} \text{ is a decreasing sequence of non-empty subsets of } X \text{ such} \\ \text{that } K_n \subset \mathrm{St}^2(x_0, \mathfrak{A}_n) \text{ for each } n \text{ and for some fixed point } x_0 \text{ of } X, \\ \text{then } \cap \bar{K}_n \neq \emptyset.\end{cases}$$

In the above definition, we may assume without loss of generality that $\{\mathfrak{A}_n\}$ is decreasing. Throughout this paper, we assume at least T_1 for every topological space unless otherwise specified.

The purpose of this paper is to show the following theorems:

(I) The image of a wM-space under a quasi-perfect map is also a wM-space (Theorem 2.1).[1]

(II) If $f: X \to Y$ is a closed continuous map of a wM-space X onto a space Y, then $Y = \bigcup_{n=0}^{\infty} Y_n$, where Y_n is discrete in Y for $n = 1, 2, \cdots$, and $f^{-1}(y)$ is countably compact for $y \in Y_0$ (Theorem 3.1).

(III) Let X be a regular space which has a sequence $\{\mathfrak{A}_n\}$ of point finite coverings of X satisfying the condition $(*)$ below:

$$(*)\begin{cases}\text{If } \{K_n\} \text{ is a decreasing sequence of non-empty subsets of } X \text{ such} \\ \text{that } K_n \text{ is contained in some } U_n \in \mathfrak{A}_n \text{ for each } n, \text{ then } \cap \bar{K}_n \neq \emptyset.\end{cases}$$

If $f: X \to Y$ is a closed continuous map of X onto a regular space Y, then $Y = \bigcup_{n=0}^{\infty} Y_n$, where Y_n is discrete in Y for $n = 1, 2, \cdots$, and $f^{-1}(y)$ is countably compact for $y \in Y_0$ (Theorem 3.2).

(II) was proved by N. Lašnev [6] for metric speces and by V. V. Filippov [3] for paracompact p-spaces (due to A. Arhangel'skii [1]), and (III) was proved by A. Arhangel'skii [2] for point-paracompact G_δ-spaces.[2] It should be noted that, in a space X with a complete structure, any closed and countably compact subset of X is compact.

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1) A quasi-perfect map $f: X \to Y$ is a closed continuous surjective map such that $f^{-1}(y)$ is countably compact for $y \in Y$.

2) Paracompact p-spaces are identical with paracompact M-spaces. Filippov [3] essentially proved (II) for M-spaces.