4. *On wM-Spaces. II*

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1. Introduction. This is the continuation of our previous paper [6]. The purpose of this paper is to study metrizability of wM-spaces and to give a solution to a problem under what conditions a wM-space is an M-space.

Definition. A topological space X has a $\bar{G}_\delta(k)$-diagonal ($G_\delta(k)$-diagonal, $k=1,2,\cdots$, if there exists a sequence $\{\mathfrak{V}_n\}$ of open coverings of X such that for distinct points x, y there exists some $\mathfrak{V}_m$ such that $y \notin \overline{\mathrm{St}^k(x,\mathfrak{V}_m)}$ $(y \notin \mathrm{St}^k(x,\mathfrak{V}_m))$.

By J. G. Ceder [5], a space X has a $G_\delta(1)$-diagonal ($=G_\delta$-diagonal in [4]) if and only if the diagonal Δ of $X\times X$ is a G_δ-subset of $X\times X$.

2. Metrizability of wM-spaces.

We shall prove some metrization theorems for wM-spaces.

Theorem 2.1. *In order that a space X be metrizable it is necessary and sufficient that X be a normal wM-space which has a $\bar{G}_\delta(1)$-diagonal.*

Proof. The necessity of the condition is obvious. To prove the sufficiency of the condition, let X be a normal wM-space with a decreasing sequence $\{\mathfrak{A}_n\}$ of open coverings of X satisfying (M_2), and suppose that X has a $\bar{G}_\delta(1)$-diagonal, that is, there exists a decreasing sequence $\{\mathfrak{V}_n\}$ of open coverings of X such that for distinct points x, y there exists some $\mathfrak{V}_n$ such that $y \notin \overline{\mathrm{St}(x,\mathfrak{V}_n)}$. Then clearly X is Hausdorff. Let us put $\mathfrak{W}_n=\mathfrak{A}_n \cap \mathfrak{V}_n, n=1,2,\cdots$. Then it is proved that $\{\mathrm{St}(x,\mathfrak{W}_n) \mid n=1,2,\cdots\}$ is a basis for neighborhoods at each point x of X. Indeed, if not, then there exist a point x_0 of X and an open subset U of X such that $x_0 \in U$ and $\mathrm{St}(x_0,\mathfrak{W}_n)-U \neq \emptyset$ for each n. Let $x_n \in \mathrm{St}(x_0,\mathfrak{W}_n)-U, n=1,2,\cdots$. Then by ($M_2$) the sequence $\{x_n\}$ has an accumulation point y which is contained in $X-U$. Since $x_0 \neq y$, we have $y \notin \overline{\mathrm{St}(x_0,\mathfrak{W}_k)}$ for some k, while $y \in \bigcap \overline{\mathrm{St}(x_0,\mathfrak{W}_n)}$. This is a contradiction, and hence $\{\mathrm{St}(x,\mathfrak{W}_n) \mid n=1,2,\cdots\}$ is a basis for neighborhoods at each point x of X. On the other hand, as is proved in our previous paper [6], every normal wW-space X is collectionwise normal (cf. [6, Theorem 2.4]). Hence, by a theorem of R. H. Bing [2], X is metrizable. Thus we complete the proof.

Theorem 2.2. *In order that a space X be metrizable it is neces-*