# 38. On the Cauchy Problem for a Certain Nonlinear Hyperbolic Partial Differential Equation of the Second Order

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**1. Introduction.** The uniqueness theorems for generalized solutions of first order quasilinear hyperbolic equations (or systems) were proved by either Holmgren's method [1], [2], or the method of using the potential function [3]-[5].

The purpose of this note is to extend the uniqueness theorems to certain second order quasilinear hyperbolic equations with two independent variables (Section 2) and with $n\,(\geqslant 2)$ independent variables (Section 3). The proofs of Lemma 1 and Theorem 1 in Section 2 are based on the potential function, and Theorem 2 in Section 3 is obtained by Holmgren's method.

In this note we state the results only. Detailed proof will be published elsewhere.

**2. The case of two independent variables.**

In $\Omega=\{a\leq x\leq b, 0\leq t\leq T, T>0\}$, we consider the following equation

(1) $$\partial^2 u(x,t)/\partial t^2=\partial A(x,t,u,\partial u/\partial x)/\partial x+B(x,t,u)$$

with initial conditions

(2) $$u(x,0)=u_0(x), \qquad \partial u(x,0)/\partial t=v_0(x)$$

where $u_0(x)\in \mathrm{Lip}\,[a,b]$ and $v_0(x)\in \mathrm{L}_\infty[a,b]$. We assume that $A(x,t,u,p)$ is of class $C^2$ with respect to all arguments and satisfies

(3) $$\partial A(x,t,u,p)/\partial p>0, \qquad \partial^2 A(x,t,u,p)/\partial p^2>0$$

and that $B(x,t,u)$ is of class $C^1$ with respect to all arguments.

The definition of the generalized solution $u(x,t)$ of the Cauchy problem (1), (2) is the following: (a) $u(x,t)\in \mathrm{Lip}\,(\Omega)$. (b) $u(x,t)$ satisfies the initial conditions (2) and the integral identity

(4) $$\oint_\Gamma u_t(x,t)dx+A(x,t,u,u_x)dt-\iint_D B(x,t,u)dxdt=0$$

where $\Gamma$ is an arbitrary piece-wise smooth closed contour, bounding a domain $D$ and lying in $\Omega$. (c) $u_x(x,t)$ possesses the semi-increasing property with respect to $t$ (in the sence of Douglis), i.e., there is a bounded measurable function $v(x,t)$ defined in $\Omega$ such that

(5) $$u_x(x,t)=v(x,t), \qquad \text{a.e. in } \Omega$$

and that