35. *Notes on Semilattices of Groups*

By Sándor LAJOS
K. Marx University of Economics, Budapest, Hungary

(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1970)

Recently a lot of ideal theoretical characterizations for semigroups which are semilattices of groups were given by the author (see [2], [3]). Continuing these investigations several further criteria will be established here. For the terminology we refer to A. H. Clifford and G. B. Preston's books [1] and for the definition of (m, n)-ideals see the author's paper [5].

Theorem 1. *An arbitrary semigroup S is a semilattice of groups if and only if the relation*

(1) $$L\cap B=LB$$

holds for any bi-ideal B and for any left ideal L of S.

Proof. *Necessity.* Let S be a semigroup which is a semilattice of groups. Then it is regular and every one-sided ideal of S is two-sided (see Exercise 4.2.2 in [1], I). In this case every bi-ideal B of S is also a two-sided ideal of S by a recent result of the author [4]. Therefore (1) follows from the well known regularity criterion:

(2) $$L\cap R=RL$$

for any left ideal L and for any right ideal R of S.

Sufficiency. Let S be a semigroup with property (1) for any left ideal L and for any bi-ideal B of S. Then (1) implies

(3) $$S\cap R=SR$$

for any right ideal R of S, and

(4) $$L\cap S=LS$$

for any left ideal L of S, that is, every one-sided ideal of S is two-sided. Thus we obtain that $A\cap B=AB$ for any two two-sided ideals A, B of S, i.e. S is regular. Next we show that S is a centric semigroup. Indeed, for any element a of S the equality (1) implies

(5) $$aS=S\cap aS=SaS,$$

and

(6) $$Sa=Sa\cap S=SaS.$$

(5) and (6) imply that $aS=Sa$ for any element a in S. It is known[1] that the idempotent elements of a centric semigroup lie in the center, thus $ef=fe$ for any two idempotent elements of S. Therefore S is an inverse semigroup every one-sided ideal of which is two-sided. This means that S is a semigroup which is a semilattice of groups.

1) See Clifford and Preston [1], II.