34. *On Locally Compact Abelian Groups with Dense Orbits under Continuous Affine Transformations*

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1. Introduction. Let G be a locally compact abelian group and let T be a continuous automorphism of G. Then the continuous affine transformation $T(a)$, where a is an element in G, is defined by $T(a)(x) = a\cdot T(x)$ for x in G. In this paper we shall study some topological properties of G which has a continuous affine transformation $T(a)$ such that there is an element w in G such that $\{T(a)^n(w) \mid n=0, \pm 1, \pm 2, \cdots\}$ is dense in G. More precisely, the study has been derived from the following problem. Can a continuous affine transformation of a locally compact but non-compact abelian group have a dense orbit? In the sequel the problem shall be solved negatively in a sense. Studies which are closely related to this problem appear in [2], [3], [4], [5] and [6].

2. Locally compact abelian groups with dense orbits.

Lemma. *Let T be a linear transformation of the n-dimensional real euclidean space R^n onto itself. Then any affine transformation $T(a)$ $(a \in R^n)$ has no dense orbit in R^n except for the trivial case $n=0$.*

Proof. T can be considered as the linear tansformation of the n-dimensional complex euclidean space K^n onto itself in the natural way. Then from the matrix theory T can be represented by a triangular matrix under some suitable basis $\{e_1, e_2, \cdots, e_n\}$ of K^n.

$$T=\begin{pmatrix} \lambda_1 & & & * \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

An elementary calculation shows that T^{-1} is also represented by the following triangular matrix under the same basis $\{e_1, e_2, \cdots, e_n\}$.

$$T^{-1}=\begin{pmatrix} \lambda_1^{-1} & & & * \\ & \lambda_2^{-1} & & \\ & & \ddots & \\ 0 & & & \lambda_n^{-1} \end{pmatrix}$$

Fix elements a and w in R^n and let

$$a=\alpha_1 e_1+\alpha_2 e_2+\cdots+\alpha_p e_p, \qquad \alpha_i \in K \text{ for } i=1,2\cdots, p$$

and

$$w=\beta_1 e_1+\beta_2 e_2+\cdots+\beta_q e_q, \qquad \beta_j \in K \text{ for } j=1,2,\cdots,q,$$