30. On Vector Measures. II

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In [4] we have proved the following theorem. *Let S be a set, R a semi-tribe (δ-ring) of subsets of S, X a normed space and m; $R \to X$ a vector measure. Then there exists a finite non-negative measure ν on R such that*

(1) *for any $A \in R$ and any number $\varepsilon > 0$ there exists a number $\delta = \delta(\varepsilon, A) > 0$ such that $B \in R$, $B \subset A$ and $\nu(B) < \delta \Rightarrow \|m(B)\| < \varepsilon$*

(2) $\nu(E) \leqq \sup \{\|m(A)\|\,;\, A \subset E,\, A \in R\}$ *for $E \in R$* ([4] *Theorem* 1).

The purpose of this paper is to point out some properties of regular vector measures by using this theorem. These properties were proved earlier (Dinculeanu [1] § 16, Theorem 3, Corollaries 1–4) for vector measures with finite variation, but we shall drop this condition and we shall consider the necessary and sufficient condition for the extension of a regular, finitely additive set function from some clan to a wider class of subsets (cf. Theorem 3). And Corollary 1 is the extension of Dinculeanu's and Kluvanek's result ([2] Theorem 5).

3. Regular vector measures. Suppose that S be a locally compact, Hausdorff space and X a Banach space.

Definition 3. Let R be a clan (ring) of subsets of S. A set function m; $R \to X$ is called regular if for every $A \in R$ and every number $\varepsilon > 0$ there exists a compact set $K \subset A$ and an open set $G \supset A$ such that for every $A' \in R$ with $K \subset A' \subset G$ we have $\|m(A) - m(A')\| < \varepsilon$.

Definition4. Let m; $R \to X$ be a set function and μ a non-negative measure on R. m is μ-absolutely continuous if for every $A \in R$ and every number $\varepsilon > 0$ there exists a number $\delta = \delta(\varepsilon, A) > 0$ such that for every $B \in R$ with $B \subset A$ and $\mu(B) < \delta$ we have $\|m(B)\| < \varepsilon$.

Lemma 2. *Let R be semi-tribe of subsets of S which has the following conditions*

(*) *for every compact set K and for every open set G such that $K \subset G$, there exists a $A \in R$ such that $K \subset A \subset G$.*

If m; $R \to X$ is a regular vector measure, then there exists a finite non-negative measure ν on R such that

(1) *m is ν-absolutely continuous.*

(2) $\nu(E) \leqq \sup \{\|m(A)\|\,;\, A \subset E,\, A \in R\}$ *for $E \in R$.*

(3) *ν is regular.*

Proof. It is easy by [4] Theorem 1.

Theorem 3. *Let R be a clan which has the following conditions*