28. *Axioms for Boolean Rings*

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G. R. Blakley, S. Ôhashi, K. Iséki and the author gave some new definitions of commutative rings and semirings (see [1]-[4]). K. Iséki gave some new axiom systems for Boolean rings (see [5]). In this note, we shall give other definitions of Boolean rings with unity.

Theorem 1. *A set with two nullary operations,* 0 *and* 1, *and with two binary operations,* + *and juxtaposition, such that*

1.1) $r+0=r$,

1.2) $r1=r$,

1.3) $(r+r)a=0$,

1.4) $(a+(br+cz))r=(br+ar)+z(cr)$

for any a, b, c, r, z, *is a Boolean ring with unity.*

Proof. We can prove this theorem as follows.

1.5) $r+r$
$=(r+r)1$ by 1.2.
$=0$ by 1.3.

1.6) $0a$
$=(0+0)a$ by 1.1.
$=0$ by 1.3.

1.7) $a+b=b+a$ (See 1.7 in [4])

1.8) $cz=zc$ (See 1.8 in [4])

1.9) $a+(b+c)=(a+b)+c$ (See 1.9 in [4])

1.10) $(zc)r=z(cr)$ (See 1.10 in [4])

1.11) $(a+c)r$
$=(a+(0r+c1))r$ by 1.6, 1.2, 1.7, 1.1.
$=(0r+ar)+1(cr)$ by 1.4.
$=ar+cr$ by 1.6, 1.7, 1.1, 1.8, 1.2.

1.12) rr
$=(0+(1r+00))r$ by 1.7, 1.8, 1.6, 1.2, 1.1.
$=(1r+0r)+0(0r)$ by 1.4.
$=r$ by 1.6, 1.1, 1.8, 1.2.

1.13) For given a, b, the equation $a+x=b$ is solvable. Let $x=a+b$. $a+(a+b)$
$=(a+a)+b$ by 1.9.
$=b$ by 1.5, 1.7, 1.1.

Hence $a+b$ is one solution of the equation.

Therefore the proof of Theorem 1 is complete.