24. *Characteristic Pseudo Quasi Topological Spaces*

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Introduction. One defines a characteristic pseudo quasi metric spaces as the topological space generated by a pseudo quasi metric function whose range is $\{0, 1\}$. Since every finite topological space is a special case of the characteristic pseudo quasi spaces, many results concerning finite topological spaces which have been known by precedents ([2], [12], [13]) are considered as the corollaries of the results of characteristic pseudo quasi metric spaces. Furthermore, every pseudo quasi metric is considered as a transformation into the reals by $f_x(y) = d(x, y)$ for each $x \in X$ and one induces an equivalent matrix representation for a finite topological space and the algebraic structure of the matrix representation is studied. Similarly, it is observed that these functions induce partial ordered relation on X.

1. This chapter is mainly concerned with necessary definitions and theorems which will be used for the discussion of the later chapters.

1.1. Definition. A $p.q.$ (pseudo quasi) metric (see [6]) "d" is said to be characteristic $p.q.$ (or $c.p.q.$) metric iff whose range is $\{0, 1\}$.

One observes $c.p.q.$ metrics act like a characteristic function on the minimum base for each $x \in X$.

For each $c.p.q.$ metrix $\tilde{d}$, there exists the conjugate $c.p.q.$ metric $\underset{\sim}{d}$, which is defined as $\underset{\sim}{d}(x, y) = \tilde{d}(y, x)$.

Notation. (1) $\tilde{S}(x, \varepsilon) = \{y : \tilde{d}(x, y) < \varepsilon, \varepsilon > 0\}$

(2) $\underset{\sim}{S}(x, \varepsilon) = \{y : \tilde{d}(x, y) < \varepsilon, \varepsilon > 0\}$

1.2. Definition. Let $\tilde{C}$ be the topology whose base is $\{\tilde{S}(x, \varepsilon)\}$ and it is said to be the characteristic topology of $\tilde{d}$. Similarly, $\underset{\sim}{C}$ is defined and $(X, \tilde{C}, \underset{\sim}{C})$ is called the $c.p.q.$ bitopological space.

The following theorem is well known ([4]-[6], [9])

1.3. Theorem. *Let the notation "$A \Rightarrow B$" be A implies B.*

$$p.q.\ \textit{bitopology} \begin{cases} \Rightarrow \textit{p-perfectly normal} \Rightarrow \textit{p-completely normal} \\ \quad \Rightarrow \textit{p-normal} \\ \Rightarrow \textit{p-completely regular} \Rightarrow \textit{p-regular}. \end{cases}$$

where "p-" denote pairwise (e.g. p-regular stands for pairwise regular)

1.4. Theorem. *Let $(X, \tilde{C}, \underset{\sim}{C})$ be a c.p.q. bitopological space.*

$$U \in \tilde{C} \text{ iff } U^c \in \underset{\sim}{C}.$$