68. A Note on Morita's P-spaces

By Vincent J. MANCUSO
St. John's University, Jamaica, New York, U. S. A.

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1970)

1. Introduction. K. Morita [10] introduced the notion of a P-space and demonstrated its importance in the theory of product spaces. The purpose of this note is to prove some results about P-spaces which will have application in homotopy extension. Let $A \subset X$ be closed and $f: X \to Y$ continuous. If, in the free union $X+Y$, we identify $a \in A$ with $f(a) \in Y$, we obtain a quotient space Z called the *adjunction space* of X and Y via the map f[3, p. 127]. A normal space X is called *totally normal* if every open subset G of X can be covered by a family locally finite in G, of open F_σ sets of X[1]. We will prove the following theorems:

Theorem 1. *If X and Y are normal P-spaces, the adjunction space Z of X and Y is a normal P-space.*

Theorem 2. *If X is a totally normal P-space and Y is a compact metric space, $X \times Y$ is a totally normal P-space.*

Actually, Theorem 2 will follow from the slightly more general:

Theorem 2′. *If X is totally normal and countably paracompact, and Y is compact metric, $X \times Y$ is totally normal.*

Theorem 3. *An open subspace of a totally normal P-space is a (normal) P-space.*

Remark 1. The compactness of Y in Theorem 2′ cannot be dropped since Michael [9] has given an example of a hereditarily paracompact (and hence totally normal and countably paracompact) space such that its product with a separable metric space is not normal. We are therefore led to the following question:

Question 1. If X is a totally normal P-space and Y is a metric space, is $X \times Y$ totally normal? Note that the normality of $X \times Y$ is assured since X is a normal P-space [10, Theorem 4.1]. In view of [11, Theorem 2], it would be sufficient to show that $X \times Y$ is hereditarily countably paracompact.

In proving Theorem 1, we will use the closed set dual of the definition of a P-space given in [10].

Definition 1. Let $\mathfrak{m}$ be a cardinal number ≥ 1. X is a *$P(\mathfrak{m})$-space* if for any set Ω of power $\mathfrak{m}$ and for any family $\{F(\alpha_1, \cdots, \alpha_i); \alpha_1, \cdots \cdots, \alpha_i \in \Omega; i=1, 2, \cdots\}$ of closed sets of X such that $F(\alpha_1, \cdots, \alpha_i) \supset F(\alpha_1, \cdots, \alpha_i, \alpha_{i+1})$ for each sequence $\alpha_1, \alpha_2, \cdots$, there exists a family