64. On Semi-inner Product Algebras*)

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1. Introduction. R. Keown [5] introduced some new classes of commutative Hilbert algebras which is some sense are generalizations of the algebras studied by W. Ambrose [1]. The essential difference between the works of Keown and Ambrose is that the latter doe not obtain the decomposition of the algebra into orthogonal subspaces each of which is a minimal left ideal. The present authors [4] generalized the work of Ambrose by replacing the underlying Hilbert space structure by a more general space called the semi-inner product space, a concept introduced by G. Lumer [6]. The purpose of this note is to extend some of Keown's results to semi-inner product spaces (henceforth abbreviated to s.i.p. spaces). For example, we show that for any generalized s.i.p. algebra A and for an idempotent e, eAe is a division algebra. For definitions we follow Keown [5] and Husain [2].

2. We recall some of the definitions from [4] and [6].

A complex (real) vector space X is called a *complex* (real) s.i.p. space if corresponding to any pair of elements $x, y \in X$, there is defined a complex (real) number $[x, y]$ which satisfies the following properties:

(i) $[x+y, z]=[x, z]+[y, z]$,
$[\lambda x, y]=\lambda[x, y]$ for $x, y, z \in X$, λ is complex or real,

(ii) $[x, x]>0$ for $x\neq 0$,

(iii) $|[x, y]|^2\leq[x, x][y, y]$.

We put $\|x\|=[x, x]^{1/2}$ and thus X is a normed space. However an s.i.p. space need *not* satisfy the following properties:

(iv) $[x, \lambda y]=\bar{\lambda}[x, y]$,

(iv)′ $[x, y]=[y, x]$

(v) $[x, y+z]=[x, y]+[x, z]$.

A s.i.p. X space is said to be *continuous* if

$$\operatorname{Re}\{[y, x+\lambda y]\}\to\operatorname{Re}\{[y, x]\} \qquad \text{for all real } \lambda\to 0,$$

and any $x, y \in X$. In a s.i.p. space X, an element $x \in X$ is said to be *orthogonal* to $y \in X$ if $[y, x]=0$. A s.i.p. space is said to be *strictly convex* if $\|x+y\|=\|x\|+\|y\|$ implies $y=\lambda x$, $\lambda>0$. An s.i.p. space which is also a Banach algebra is said to be a *generalized s.i.p. algebra*. A generalized s.i.p. algebra A is said to be *regular* if corresponding

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