63. *Asymptotic Property of Solutions of Some Higher Order Hyperbolic Equations. II*

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3. In this part, we consider the inhomogeneous equation

$$(2)' \qquad \prod_{j=1}^{m} [\partial_t^2 + \alpha_j L]u(t) = ge^{i\omega t},$$

where $g \in X$ and $\omega \neq 0$ real. We restrict ourselves to the case when the Hilbert space X and the operator $H = L^{1/2}$ satisfy the following conditions, and prove the so called limiting amplitude principle.

[C.1] There exists a Fréchet space Y, into which X is densely injected, with semi-norms $\{\rho_\nu(f) = [\rho_\nu(f, f)]^{1/2};\ \nu = 1, 2, \cdots\}$ having the following properties:

$$(28) \qquad \rho_\nu(f) \leq \rho_{\nu+1}(f) \leq \|f\| \text{ and } \sup_\nu \rho_\nu(f) = \|f\| \text{ for all } f \in X.$$

[C.2] The set X' defined below is dense in X.

Definition. We denote by X' the set of all $g \in X$ which satisfy the following two conditions:

(i) Let $[a, b]$ be any bounded interval in $\boldsymbol{R}_+^1$. Then, as $\varepsilon \to \pm 0$, $(H - \sigma - i\varepsilon)^{-1}g$ converges uniformly in $\sigma \in [a, b]$ in the sense of each ρ_ν-topology.

(ii) We put $(H - \sigma \mp i0)^{-1}g \equiv \lim_{\varepsilon \to \pm 0} (H - \sigma - i\varepsilon)^{-1}g$. Then $(H - \sigma \mp i0)^{-1}g$ is a Hölder continuous function of $\sigma \in \boldsymbol{R}_+^1$ with values in Y.

[C.3] The origin 0 is not an eigenvalue of H.

Now, by the same reasoning as in the proof of Theorem 3, we see that the initial value problem (2)′, (3) has a unique solution in the class $\bigcap_{0 \leq j \leq 2m} \mathcal{E}_t^j(D(H^{2m-j+1}))$. Further, it follows that

$$(29) \qquad H^{2m-j}\partial_t^{j-1}u(t) = \sum_{k=1}^{2m} (\gamma_k)^{j-1} e^{\gamma_k H t} \sum_{l=1}^{2m} n_{kl} H^{2m-l} \varphi_l + \sum_{k=1}^{2m} (\gamma_k)^{j-1} \int_0^t e^{\gamma_k H(t-s)} n_{k2m} g e^{i\omega s} ds$$

(cf., (26)).

Lemma 3. *If we choose $g \in X'$, then as $t \to \infty$*

$$(30) \qquad H^{2m-j}\partial_t^{j-1}u(t) \to ie^{i\omega t} \sum_{k=1}^{2m} (\gamma_k)^{j-1}(-i\gamma_k H - \omega + i0)^{-1} n_{k2m} g$$

in the sense of each ρ_ν-topology.

Proof. Note that for any $\gamma \neq 0$ pure imaginary and $f \in X$,

$$e^{\gamma H t} f = \int_0^\infty e^{\gamma \sigma t} dE_\sigma^H f$$