62. Asymptotic Property of Solutions of Some Higher Order Hyperbolic Equations. I

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Introduction. Let X be a complex Hilbert space with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$. Let L be a selfadjoint (in general unbounded) operator on X satisfying

(1) $$(Lf, f)\geq 0 \quad \text{for all } f\in\mathcal{D}(L),$$

where $\mathcal{D}(L)$ denotes the domain of L. We shall consider abstract "hyperbolic" equations of the form

(2) $$\prod_{j=1}^{m}[\partial_t^2+\alpha_j L]u(t)=0 \ (t\in \boldsymbol{R}^1)$$

$(\partial_t=d/dt)$ with initial data

(3) $$\partial_t^{j-1}u|_{t=0}=\varphi_j\in\mathcal{D}(L^{(2m-j+1)/2}),\ j=1,2,\cdots,2m,$$

where m is a positive integer and α_j are positive constants such that

(4) $$0<\alpha_1<\alpha_2<\cdots<\alpha_m.$$

In Mizohata [2], we know that there exists a unique solution of (2), (3) in the class $\bigcap_{0\leq j\leq 2m}\mathcal{E}_t^j(\mathcal{D}(L^{(2m-j)/2}))$[1] ([2]; Theorem 5.1). In this note, we shall obtain an asymptotic property as $t\to\infty$ of the solution under the assumption that the spectrum of L is strongly absolutely continuous with respect to the Lebesgue measure. As will be seen, we shall generalize recent results of Shinbrot [4] and Goldstein [1], in which are treated the case of abstract wave equations (i.e., when $m=1$ in (2)).

First we consider the case when the origin 0 is in the resolvent set of L. In this case, applying the method developed by Mizohata [2], we can construct the explicit formula of the strongly continuous group $\{T_t\,;\,t\in\boldsymbol{R}^1\}$ of unitary operators in the space $\prod_{j=1}^{2m}\mathcal{D}(L^{(2m-j)/2})$ which assign to given initial data $(\varphi_1,\varphi_2,\cdots,\varphi_{2m})$ the data of corresponding solution of (2) at time t. For the general case, let $L_n=L+2n^{-1}L^{1/2}+n^{-2}I$. Then, by the limit procedure developed by Goldstein [1], we can deduce the general case from the special case that L is invertible.

1. Assume first that there exists a positive constant c such that

(5) $$(Lf, f)\geq c\|f\|^2 \quad \text{for all } f\in\mathcal{D}(L).$$

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1) $u(t)\in\mathcal{E}_t^j(X)$ means that $u(t)$ is j times continuously differentiable in t with values in X.