52. An Estimate from above for the Entropy and the Topological Entropy of a C^1-diffeomorphism

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Let φ be a C^1-diffeomorphism from an n-dimensional Riemannian manifold on itself, $h(\varphi)$ the topological entropy [1] of φ and let λ be a contractive constant of φ. In this paper, we will give an estimate from above for the topological entropy:

$$h(\varphi)\leqslant n\log 1/\lambda$$

Using a result of L. Goodwyn [3], one can derive also an estimate from above for the measure theoretic entropy [7]:

$$h_\mu(\varphi)\leqslant n\log 1/\lambda$$

and this estimate is sharper than Kuchinirenko's [6] and A. Avez's [2].

§1. Definitions and a property.

Let φ be a homeomorphism from a compact metric space X onto itself. If α is any open cover of X, we let $N(\alpha)$ be the number of members in a subcover of α of minimal cardinality. As in [1], the limit exists in the following definition:

$$h(\alpha,\varphi)=\lim_{m\to\infty}\frac{1}{m}\log N(V_{i=0}^{m-1}\,\varphi^i\alpha)^{*)}$$

Let α_t be the collection of all open spheres of radius $t>0$. In metric spaces, the topological entropy $h(\varphi)$ of φ can be defined as $h(\varphi)=\lim_{t\to 0} h(\alpha_t,\varphi)$. (This is equivalent to the usual definition.)

For any $t>0$, let β_t be any cover of subset A of X by arbitrary sets of diameter$\leqslant 2t$.

For any set A of X, define $M_t(A)$ to be the number of members in subcover of β_t of minimal cardinality. Then as in [5], we define the lower metrical dimension $\underline{\dim}\,A$ of set A by

$$\underline{\dim}\,A=\underline{\lim}_{t\to 0}\frac{\log M_t(A)}{\log 1/t}$$

and define the dimension $\dim A$ of set A by

$$\dim A=\lim_{t\to 0}\frac{\log M_t(A)}{\log 1/t}\quad \text{if the limit}$$

exists.

Property 1 [5]. *Let X be an n-dimensional Euclidian space and suppose a compact subset A of X has interior points.*

*) As in [1], we write $\alpha\vee\beta=\{U\cap V\colon U\in\alpha,\ V\in\beta\}$ and we write $\alpha>\beta$ to mean that α is a refinement of β.