88. *The Existence and Uniqueness of the Solution of the Equations Describing Compressible Viscous Fluid Flow*

By Nobutoshi ITAYA

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Since J. Leray's discussion on the nonstationary movement of incompressible viscous fluid, there appeared a number of papers on it such as Kiselev-Ladyzhenskaya's ([1]), but very few reports, if any, on that of compressible viscous fluid have been made, presumably, because of the complexities that the system of equations describing it contains. In view of these circumstances, we try to find a way of solving this problem firstly from a classical point of view.

1. Introduction. When μ(viscosity), k(heat conductivity), and c_v(specific heat at constant volume) are constants (which does not injure the mathematical generality), the movement of isotropic Newtonian fluid is described as follows: (ρ: density, v: velocity, f: outer force, p: pressure, θ: absolute temperature, and $F(\nabla v)$: dissipation function (≥ 0)),

$$(1.1)\quad \begin{cases} \dfrac{\partial \rho}{\partial t}+\operatorname{div}\rho v=0, \\ \rho\left(\dfrac{\partial v}{\partial t}+(v\cdot\nabla)v\right)=\rho f-\nabla\rho\dfrac{\partial p}{\partial \rho}-\nabla\theta\dfrac{\partial p}{\partial\theta}+\left(\mu\Delta+\dfrac{\mu}{3}\cdot\nabla\operatorname{div}\right)v, \\ c_v\rho\dfrac{\partial\theta}{\partial t}=k\Delta\theta+F-\theta\dfrac{\partial p}{\partial\theta}\operatorname{div}\rho v-c_v\rho v\cdot\nabla\theta, \end{cases}$$

$$(1.1)'\quad p=\theta\sum_{n=1}^{\infty}\hat{a}_n\rho^n,\ (0<\rho<\rho^*=\text{radius of convergence of } p;\ p \text{ is assumed to be virially expanded}).$$

We shall consider a Cauchy problem of (1.1) in which the initial condition is given by

$$(1.1)''\quad \rho(x,0)=\rho_0(x)(>0),\ v(x,0)=v_0(x),\ \theta(x,0)=\theta_0(x)(\geq 0).$$

In the first place, we make the following linear problem correspond with the 2nd expression of (1.1).

$$(1.2)\quad \begin{cases} \dfrac{\partial v}{\partial t}=\sigma(x,t)\left(\Delta+\dfrac{1}{3}\nabla\operatorname{div}\right)v+f\equiv\sigma(x,t)P_0(D_x)v+f, \\ v(x,0)=v_0(x)(\in H^{2+\alpha}(\overline{R^3})),\ ((x,t)\in R^3_T), \end{cases}$$

where σ, $f\in H^\alpha_T$, $0<\sigma_0\leq\sigma(x,t)\leq\sigma_1<+\infty$, and H^α_T is the space of functions $g(x,t)$ defined on $\overline{R^3_T}(R^3_T\equiv R^3\times(0,T))$ such that