86. *Connection of Topological Manifolds*

By Akira ASADA
Department of Mathematics, Shinsyu University, Matumoto

(Comm. by Kinjirô KUNUGI, M. J. A., April 13, 1970)

Introduction. The notion of connection for topological fibre bundles has been introduced by the author ([1], [2]). Then a connection of a topological manifold X should be defined to be a connection of the tangent microbundle ([10]) of X. The purpose of this note is to show the existence of connection for any paracompact topological manifold and state some related topics. The details will appear in the Journal of the Faculty of Science, Shinshu University, Vol. 5, 1970.

1. Connection of topological fibre bundles. Let $\xi=\{g_{UV}(x)\}$ be a topological G-bundle over a normal paracompact space X, where G is a topological group, $\{g_{UV}(x)\}$ is the transition function of ξ with covering system $\{U\}$. Then a connection $\theta=\{s_U(x_0, x_1)\}$ of ξ is a collection of the germ (at the diagonal of $U\times U$) of G-valued function $s_U(x_0, x_1)$ such that

$$S_U(x, x)=e, \textit{ the unit of } G,$$
$$g_{UV}(x_0)s_V(x_0, x_1)g_{VU}(x_1)=s_U(x_0, x_1).$$

We denote by $\mathcal{G}=\mathcal{G}_G$ the sheaf of germs of the germ (at the diagonal of $X\times X$) of G-valued function $\{t_U(x_0, x_1)\}$ such that

$$t_U(x, x)=e,$$
$$g_{UV}(x_1)t_V(x_0, x_1)g_{VU}(x_1)=t_U(x_0, x_1),$$

then we can define a cohomology class $o(\xi)$ of $H^1(X, \mathcal{G})$ such that ξ has a connection if and only if $o(\xi)$ vanishes in $H^1(X, \mathcal{G})$ ([3]).

In fact, if G is either of

(i) *There is a topological ring $R\supset G$ such that there is a neighbourhood $U(e)$ of e in R which is contained in G,*

(ii) *G is a locally compact, connected, locally connected solvable group,*

then a G-bundle ξ has a connection ([1], [3]).

If $\theta=\{s_U(x_0, x_1)$ is a connection of ξ, then

$$\delta\theta=\{s_U(x_1, x_2)s_U(x_0, x_2)^{-1}s_U(x_0, x_1)\}$$

is called the curvature of θ. We can prove that if the value of $\delta\theta$ is contained in H, a subgroup of G, then the connected component of the structure group of ξ is reduced to H ([1], [2]).

Note 1. If $G=\boldsymbol{C}^*$, the multiplicative group of complex numbers without 0, then the Alexander-Spanier class of $\delta\theta$ is the 1-st (complex)