85. Other Characterizations and Weak Sum Theorems for Metric-dependent Dimension Functions

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1. Introduction. In [7] and [8] the author introduced the metric-dependent dimension functions d_6 and d_7 and characterized them in terms of Lebesgue covers of metric spaces and for uniform spaces. The results for metric spaces are the following.

Theorem 1.1. *Let (X, ρ) be a metric space. Then $d_6(X, \rho) \leq n$ if and only if every countable Lebesgue cover has an open refinement of order $\leq n+1$.*

Theorem 1.2. *Let (X, ρ) be a metric space. Then $d_7(X, \rho) \leq n$ if and only if every locally finite Lebesgue cover of X has an open refinement of order $\leq n+1$.*

A natural question now arises as to whether new metric-dependent dimension functions occur if "countable" and "locally finite" in the above characterization theorems are replaced by "star-countable" and "point finite" respectively. In §2 we define two such new dimension functions, d_6^* and d_7^*, and prove that $d_6^* = d_6$ and $d_7^* = d_7$. We also show that the dimension function d_5 of Hodel [1] has a "star-countable" equivalent definition. In §3 we introduce a new metric-dependent dimension function d_3^*, characterize it in terms of Lebesgue covers, and observe the following inequality $d_3 \leq d_3^* \leq d_6$. In §4 we generalize a sum theorem of Morita and establish "weak" locally finite sum theorems for $d_2, d_3, d_3^*, d_6, d_7$ and d_0 in both metric and uniform spaces.

2. Equivalent characterization for d_6 and d_7.

Definition 2.1. Let (X, ρ) be a metric space. Then $d_6^*(X, \rho) \leq n$ if and only if every star-countable Lebesgue cover of X has an open refinement of order $\leq n+1$.

We note that $d_6(X, \rho) \leq d_6^*(X, \rho)$ by Definition 2.1 and Theorem 1.1. By a similar technique as in Theorem 2 of [2] by Morita we have the following.

Theorem 2.2. *Let $\mathcal{G} = \{G_\alpha : \alpha \in A\}$ be a star-countable open cover of a T_1 space X. We divide the index set A into subsets $\{A_\beta : \beta \in B\}$ such that α and γ belong to A_β if and only if there exists a positive integer n such that $G_\alpha \subset \mathrm{St}^n(G_\gamma, \mathcal{G})$. Define $X_\beta = \bigcup_{\alpha \in A_\beta} G_\alpha$. Then we have the following*

(1) $X = \bigcup_{\beta \in B} X_\beta$