79. On the Existence of a Potential Theoretic Measure with Infinite Norm

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Introduction. Let R^m be the m-dimensional Euclidian space and $\phi(x, y)$ a lower semi-continuous function from $R^m \times R^m$ into $[0, +\infty]$. The ϕ-potential of a positive Radon measure μ in R^m is defined by

$$\phi\mu(x)=\int\phi(x, y)d\mu(y).$$

In the case that there exists at least such a positive measure ν that the support $S\nu$ is compact and the potential $\phi\nu(x)$ is continuous in the whole space R^m, we can consider the following classes of measures;

$$\mathcal{F}(\phi)=\{\nu\,;\nu\geqq 0,\ S\nu \textit{ compact and } \phi\nu(x) \textit{ continuous in } R^m\},$$

$$\mathcal{G}(\phi)=\left\{\mu\,;\mu\geqq 0 \textit{ and } \int\phi\mu d\nu<+\infty \textit{ for any } \nu\in\mathcal{F}(\phi)\right\}.$$

The aim of this paper is to answer affirmatively for a question posed by G. Anger [1]: Let $\phi_N(x, y)$ be the Newtonian kernel defined in R^m $(m\geqq 3)$. Is there a measure $\mu\in\mathcal{G}(\phi_N)$ with infinite norm? Moreover we study the same problem in case of α-kernel $\phi_\alpha(x, y)$.

1. Existence of a measure $\mu\in\mathcal{G}(\phi_N)$ with infinite norm.

The Newtonian kernel $\phi_N(x, y)$ in R^m $(m\geqq 3)$ is defined by

$$\phi_N(x, y)=|x-y|^{2-m},$$

where $|x-y|$ denotes the distance between two points x and y in R^m. Let $B_{a,r}$ be the closed ball with the center a and the radius r and $S_{a,r}$ the surface of the ball $B_{a,r}$. We introduce the class of measures

$$\mathcal{S}=\{\lambda\,; \textit{spherical distribution with uniform density}\}.$$

Especially the spherical distribution with uniform density on $S_{a,r}$ is denoted by $\lambda_{a,r}$. It is well known that $\mathcal{S}$ is a non empty subset of $\mathcal{F}(\phi_N)$. Let us recall following potential theoretic principles,

Maximum principle: If it holds that, for a constant V, $\phi\nu(x)\leqq V$ on the support $S\nu$ of a positive measure ν, then we have the same inequality in the whole space.

Domination principle: If it holds that, for a positive measure ν and an energy finite positive measure μ, $\phi\mu(x)\leqq\phi\nu(x)$ on the support $S\mu$, then we have the same inequality in the whole space.

Lemma 1. *For a given positive measure μ, the mutual energy $\int\phi_N\mu d\nu$ is finite for any $\nu\in\mathcal{F}(\phi_N)$ if $\int\phi_N\mu d\lambda$ is finite for any $\lambda\in\mathcal{S}$.*