156. *Approximation of Obstacles by High Potentials; Convergence of Scattering Waves*

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1. Introduction and summary. In this paper it is shown that the solutions (the outgoing scattering waves) of the Dirichlet zero boundary value problem for the equation

$$(-\Delta+q(x))\varphi=|k|^2\varphi \tag{1.1}$$

in an exterior domain in 3-space is approximated in a certain sense by the solutions φ_n of the equations in the whole space

$$(-\Delta+q(x)+n\chi_K(x))\varphi_n=|k|^2\varphi_n, \qquad n=1,2,\cdots. \tag{1.2}$$

Here K is an obstacle and χ_K denotes the characteristic function of K.

The same problem for discrete eigenvalues was considered by K. Ōeda [5] and he showed the convergence of negative eigenvalues and eigenfunctions. There are some other convergence problems of this type. For instance, P. Werner [7] proved that the scattering waves for the exterior Neumann problem can be approximated by the waves in the whole space when the damping factor is made large in the obstacle.

Let K be a compact set $\subset R^3$. We suppose that ∂K, the boundary of K, is a closed surface of class C^2, and that the complement of K is connected. The potential function $q(x)$ is assumed to be a real-valued, Hölder continuous function of $x\in R^3$, satisfying the inequality $|q(x)|\leqq C|x|^{-3-\delta}$ for $|x|\geqq R_0$ with positive constants C,δ and R_0. The non-zero 3-vector k is fixed throughout this paper.

In what follows, we denote by $H^m(\Omega)$ the Sobolev space of order m over a domain Ω, by $\| \ \|_{m,\Omega}$ the norm in $H^m(\Omega)$ and by $H^m_{\mathrm{loc}}(\Omega)$ the local space. Furthermore, let us denote by $H^m_0(\Omega)$ the closure of $C^\infty_0(\Omega)$ in $H^m(\Omega)$.

Problem (I). *Find a function $\varphi(x)$ on R^3-K which satisfies the following conditions*: i) $\zeta\varphi\in H^2(R^3-K)\cap H^1_0(R^3-K)$ *where $\zeta(x)$ is an arbitrary function in $C^\infty(R^3-K)$ with $\zeta(x)\equiv 1$ near ∂K and $\zeta(x)\equiv 0$ for large* $|x|$[1]; ii) $\varphi(x)$ *satisfies* (1.1) *in the sense of distributions*; iii) $v(x)\equiv\varphi(x)-e^{ik\cdot x}$ *satisfies the radiation condition at infinity*

$$v(x)=O(|x|^{-1}), \quad (\partial/\partial|x|-i|k|)v(x)=o(|x|^{-1}) \quad (|x|\to\infty) \tag{1.3}$$

uniformly with respect to every direction.

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1) Under these assumptions, however, φ becomes the classical solution. That is, φ is twice continuously differentiable in R^3-K and continuous up to the boundary.