155. *Approximation of Obstacles by High Potentials; Convergence of Eigenvalues*

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§ 1. Introduction.

Let K be a compact subset of R^3 whose boundary is of class C^2 and $\Omega = R^3 - K$. Consider the following equation of Schrödinger type in Ω with the Dirichlet boundary condition:

$$(1)\qquad \begin{cases} -\Delta\varphi(x) + q(x)\varphi(x) = \lambda\varphi(x), \\ \varphi(x)\,|_{\partial K} = 0. \end{cases}$$

Furthermore, let us consider the Schrödinger equation of the form

$$(2)\qquad -\Delta\varphi(x) + q(x)\varphi(x) + n\chi_K(x)\varphi(x) = \lambda\varphi(x)$$

in the whole space R^3, where $\chi_K(x)$ is the characteristic function of K and n is a positive integer.

The purpose of the present paper is to show that the negative eigenvalues of (1) can be obtained as a limit of those of (2) when n tends to infinity. Convergence of eigenfunctions will also be discussed.

The idea of regarding (1) as the limit problem of (2) is closely related to the penalty method (cf. Lions [3]). It may be noted that χ_K in (2) can be replaced by any function f which is measurable, positive and bounded on K and is zero outside K. In a physical sense Problem (1) is sometimes referred to as the hard core model. Thus, as far as eigenvalues and eigenfunctions are concerned, the hard core, i.e. the infinite potential on K, can be approximated by potentials which are strongly repulsive on K. Furthermore, looking in the reverse way, one may use the hard core to approximate such a potential on K.

Among related works we mention those of Titchmarsh [6] and Konno [2]. Titchmarsh obtained the eigenfunction expansions for a finite two-dimensional region by making $q(x) \to \infty$ outside the region considered. Recently Konno considered the same problem as ours and proved the convergence of eigenfunctions belonging to the continuous spectrum.

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§ 2. Statement of results.

Throughout the present paper we always assume that $q(x)$, a real