148. *Ergodic Automorphisms and Affine Transformations*

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By modifying an argument which originated with the authors R. Sato [6] has attempted to prove that *if T is a continuous affine transformation of a locally compact group G such that the orbit $\{T^n(x): -\infty<n<\infty\}$ of some element x is dense in G, then G is compact.* That paper was suggested by a series of papers aimed at answering the following question first raised by Halmos [2, p. 29]. Can an automorphism of a locally compact but non-compact group be an ergodic measure-preserving transformation?

As pointed out in [4], the stated key lemma (Lemma 1) of the argument in [6] does not in fact hold. The question of Halmos, as well as the results stated in [6] and [7, I, Theorem 3], thus remain open questions. The purpose of this note is to announce, in § 1, some results bearing on the question of Halmos and on the analogous one for affine transformations. The proofs will appear in [5]. In § 2 we answer in the affirmative a question raised by Sato in [7, II].

1. Groups with ergodic transformations. Although the theorems below are stated in the context of ergodic, measure-preserving transformations, all the results remain valid if ergodicity is replaced by the assumption that the orbit of some element is dense. G will denote a locally compact group and G_0 its identity component. By an affine transformation on G we mean a mapping of the form $T(x)=a\tau(x)$, where τ is a bi-continuous automorphism of G and $a\in G$.

Theorem. *Let G have an ergodic automorphism. If G/G_0 is compact, then G must be compact. Thus if there exists a noncompact group with an ergodic automorphism then there exists a noncompact totally disconnected one.*

Theorem. *Let G be totally disconnected, and assume it has an ergodic automorphism. If G also satisfies one of the following conditions, then it must be compact:*

(i) *Every compact subset of G is contained in a compact subgroup.*

(ii) *G has a compact, open, normal subgroup.*

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