70. On the Minimal Group Congruence on the Tensor Product of Archimedean Commutative Semigroups

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By the tensor product $X\otimes Y$ of commutative semigroups X and Y we mean the quotient semigroup $F(X\times Y)/\delta$ where $F(X\times Y)$ is the free commutative semigroup on the set $X\times Y$ and δ is the smallest congruence relation for which:

$$(x_1+x_2, y)\delta(x_1, y)+(x_2, y)$$

and

$$(x, y_1+y_2)\delta(x, y_1)+(x, y_2)$$

hold for all $x_1, x_2, x\in X$ and $y, y_1, y_2\in Y$.

If α and β are congruences on semigroups X and Y, then $\alpha\otimes\beta$, which is called the tensor product of congruences α and β, is the smallest congruence on the tensor product $X\otimes Y$ containing all pairs $(x_1\otimes y_1, x_2\otimes y_2)$ such that

$$(x_1, x_2)\in\alpha \quad\text{and}\quad (y_1, y_2)\in\beta, \qquad \text{(see, [2]).}$$

A congruence δ on a semigroup X is called a group congruence if X/δ is a group. W. D. Munn [4] proved that a relation α defined on an inverse semigroup X by the rule that $x_1\ \alpha\ x_2$ $(x_1, x_2\in X)$ if and only if $x_1+e=x_2+e$ for some idempotent e of X is the minimal group congruence on X. The author [3] proved that X and Y are commutative inverse semigroups which possess the minimal group congruences α and β, respectively, then the tensor product $X\otimes Y$ possesses the minimal group congruence and it is the tensor product $\alpha\otimes\beta$. In this note we shall give such a property in the case when X and Y are archimedean commutative semigroups with idempotents, where a commutative semigroup X is called archimedean if for every $a, b\in X$, there exist elements $x, y\in X$ and positive integers m, n such that

$$ma=b+x \quad\text{and}\quad nb=a+y, \qquad \text{(see, [5] or [1]).}$$

Lemma 1 ([5] Theorem 3). *An archimedean commutative semigroup can contain at most one idempotent.*

Lemma 2. *Let X be an archimedean commutative semigroup with an idempotent e and let a relation α be defined on X by the rule that $x_1\ \alpha\ x_2$ $(x_1, x_2\in X)$ if and only if*

$$x_1+e=x_2+e.$$

Then α is a congruence and X/α is a group. Further, if γ is any con-