58. *Prime Ideals in the Dual Objects of Locally Compact Groups*

By Nobuhiko TATSUUMA
Department of Mathematics, Kyôto University

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1971)

1. Let G be a locally compact group, and Ω be the set of equivalence classes of unitary representations of G, dimensions of which are lower than a sufficiently large fixed cardinal number (for instance the large one of countable infinite or dim $L^2(G)$). Then we can introduce a product operation $\otimes$ in Ω by the Kronecker product of representations, and the addition operation $\oplus$ in Ω by the direct sum of representations (We allow infinite discrete direct sum). So that, a ring-like structure is given in Ω.

Now we shall call a subset $\mathfrak{J}$ an *ideal* in Ω when

i) *$\mathfrak{J}$ is closed with respect to the operation $\oplus$.*

ii) *If ω is in $\mathfrak{J}$ then any subrepresentation* of ω is in $\mathfrak{J}$.

iii) *For any ω in $\mathfrak{J}$ and any ω_0 in Ω, $\omega_0 \otimes \omega$ is in $\mathfrak{J}$.*

Moreover, we shall call an ideal $\mathfrak{J}$ in Ω is *prime* when

iv) *If ω_1, ω_2 are both disjoint to any representations in $\mathfrak{J}$, in the sense of G. W. Mackey* [1], *then $\omega_1 \otimes \omega_2$ too.*

As is well-known, Kronecker product of any ω in Ω and the regular representation $\mathfrak{R}$ is unitary equivalent to a multiple of $\mathfrak{R}$. So that, the set $\mathfrak{J}_{\mathfrak{R}}$ of classes of all subrepresentations of multiples of $\mathfrak{R}$ gives the smallest non-empty (but in general not prime) ideal in Ω.

On the other hand, in the previous paper [2], we gave examples of non-trivial operator fields $\{T(\omega)\}$ over Ω which commute with the both of operations $\otimes$ and $\oplus$, and $T(\mathfrak{R})=0$ (p. 225, Example 3 and p. 226, Example 5). There exists close connection between such an operator field and non-trivial prime ideal.

The purpose of this paper is to show this connection, and to give an example of non-trivial prime ideal in Ω as an extension of the examples in the paper [2]. And this leads to a new proof of that every unitary irreducible representations of compact group are finite dimensional.

2. Now we shall give the correspondence between non-trivial prime ideals in Ω and a family of non-zero operator fields $\{T(\omega)\}$ over Ω which commute with the both of operations $\otimes$ and $\oplus$ and $T(\mathfrak{R})=0$, under the additional condition, that $T(\omega)^{-1}(0)$ is G-invariant for any ω in Ω.