57. A Geometrical Method for Optimal Control Problem for Some Non-linear Systems

By Toshio NIWA

(Comm. by Kinjirô KUNUGI, M. J. A., March 12, 1971)

0. Introduction. In this note we study the problem of optimal control for some non-linear systems.

Let us consider the following control system:

$$\frac{dx}{dt}=f(x,u), \tag{1}$$

where u is a control parameter and belongs to some control domain U. As was shown by E. Roxin [1] and J. Warga [3], it is proper to assume the set $F(x)=\{f(x,u)\,;u\in U\}$ is compact and convex. In fact convexity of $F(x)$ implies the closedness of the reachable set of the system (1), therefore it guarantees the existence of optimal control for most control systems, at least for time-optimal problem. Moreover, for the general control system,

$$\frac{dx}{dt}\in G(x), \tag{2}$$

if we take its relaxed system (3) instead of (2),

$$\frac{dx}{dt}\in \text{Convex hull of } G(x) \tag{3}$$

then for any solution $x(t)$ of (3) there exists a solution of (2) which approximates $x(t)$ uniformely under fairly general condition, and consequently it will be proper to consider the system (3) in the place of (2).

For the simplicity we consider the time-optimal problem and assume $F(x)$ is a compact convex set generated by finitely many extremal points (vectors).

In the problem of time-optimal control, the value $f(x,u)$ itself is more important than the one of control parameter u, so we set the system (1) in the following form:

$$\frac{dx}{dt}\in \text{Convex}\,\{X_1(x),\cdots,X_r(x)\}, \tag{4}$$

where x denotes a point of $R^n, X_i(x)$ $(i=1,\cdots,r)$ smooth vector fields on R^n, and Convex $\{X_1(x),\cdots,X_r(x)\}$ the convex set generated by the points (vectors) $X_1(x),\cdots,X_r(x)$.

1. Definitions. $x=x(t)=x(t\,;x_0,t_0)(x(t_0\,;x_0,t_0)=x_0)$ is said to be an admissible trajectory of the control system (4), when it is piece-wise smooth and satisfies the following relation: