56. *Remarks on the Eichler Cohomology of Kleinian Groups*

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1. Let Γ be a finitely generated kleinian group, Ω its region of discontinuity, Λ its limit set and $\lambda(z)|dz|$ the Poincaré metric on Ω. We denote by Δ an arbitrary Γ-invariant union of components of Ω. In this note we assume that Δ/Γ is a finite union of compact Riemann surfaces, and consider relations between the Kra and the Ahlfors decompositions for $H^1(\Gamma, \Pi_{2q-2})$.

2. We fix an integer $q\geqq 2$. Let Ξ be an Γ-module. A mapping $p: \Gamma\to\Xi$ is called Ξ-cocycle if $p_{AB}=p_A\cdot B+p_B$, $A, B\in\Gamma$. If $f\in\Xi$, its coboundary δf is the cocycle $A\to f\cdot A-f$, $A\in\Gamma$. The first cohomology space $H^1(\Gamma,\Xi)$ is the space of cocycles factored by the space of coboundaries. The Γ-modules used in this note are (1) Π_{2q-2}, the vector space of complex polynomials in one variable of degree at most $2q-2$, with $v\cdot A(z)=v(Az)A'(z)^{1-q}$, $v\in\Pi_{2q-2}$ and $A\in\Gamma$ and (2) $\boldsymbol{H}_r(\Delta)(\boldsymbol{M}_r(\Delta))$ the vector space of holomorphic (meromorphic) functions on Δ, with $f\cdot A(z)=f(Az)A'(z)^{1-q}$, $f\in \boldsymbol{H}_r(\Delta)(\boldsymbol{M}_r(\Delta))$, $A\in\Gamma$, where r is an integer. We call $\boldsymbol{H}_r(\Delta,\Gamma)$ and $\boldsymbol{M}_r(\Delta,\Gamma)$, the spaces of holomorphic and meromorphic automorphic forms of weight $(-2r)$ on Δ for Γ, respectively. Two meromorphic (holomorphic) Eichler integrals of order $1-q$ are identified if they differ an element of Π_{2q-2}. This identification space is denoted by $\boldsymbol{E}_{1-q}(\Delta,\Gamma)(\boldsymbol{E}^0_{1-q}(\Delta,\Gamma))$. If $a_1, a_2, \cdots, a_{2q-1}$ are distinct points in Λ and $\phi\in\boldsymbol{H}_q(\Delta,\Gamma)$, then

$$F(z)=\frac{(z-a_1)\cdots(z-a_{2q-1})}{2\pi i}\iint_\Omega \frac{\lambda^{2-2q}(\zeta)\bar{\phi}(\zeta)d\zeta\wedge d\bar{\zeta}}{(\zeta-z)(\zeta-a_1)\cdots(\zeta-a_{2q-1})}$$

is a potential for ϕ (Bers [2]). We denote by $\mathrm{Pot}(\phi)$ a potential for ϕ. A mapping $\alpha: \boldsymbol{E}^0_{1-q}(\Delta,\Gamma)\to H^1(\Gamma,\Pi_{2q-2})$ is defined as $\alpha_A(f)=f\cdot A-f$ for $f\in\boldsymbol{E}^0_{1-q}(\Delta,\Gamma)$ and $A\in\Gamma$. A mapping $\beta^*: \boldsymbol{H}_q(\Delta,\Gamma)\to H^1(\Gamma,\Pi_{2q-2})$ is defined by setting $\beta^*_A(\phi)=\mathrm{Pot}(\phi)\cdot A-\mathrm{Pot}(\phi)$ for $\phi\in\boldsymbol{H}_q(\Delta,\Gamma)$.

Theorem A (The Kra decomposition). *Every $p\in H^1(\Gamma,\Pi_{2q-2})$ can be written uniquely as $p=\alpha(f)+\beta^*(\phi)$ with $f\in\boldsymbol{E}^0_{1-q}(\Delta,\Gamma)$ and $\phi\in\boldsymbol{H}_q(\Delta,\Gamma)$.*

3. For $f\in\boldsymbol{E}_{1-q}(\Delta,\Gamma)$, the polynomials $f(Az)A'(z)^{1-q}-f(z)$ are the periods of f, and we write $f(Az)A'(z)^{1-q}-f(z)=pd_Af(z)$. The periods determine a canonical isomorphism $pd: \boldsymbol{E}_{1-q}(\Delta,\Gamma)\to H^1(\Gamma,\Pi_{2q-2})$. Thus pdf, $f\in\boldsymbol{E}_{1-q}(\Delta,\Gamma)$, is a cohomology class and $pd\boldsymbol{E}_{1-q}(\Delta,\Gamma)$ is the image