88. A New Characterization of Regular Duo Semigroups

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Let S be a semigroup. Following the notation and terminology of A. H. Clifford and G. B. Preston [1] we say that S is regular if, to every element a in S, there exists at least one element x in S such that $axa=a$. For the sake of brevity we shall say that S is a duo semigroup if every one-sided ideal of S is two-sided. The author proved several ideal-theoretic characterizations of regular duo semigroups (cf. [2]-[4]).

In this short note some new criteria for a semigroup to be a regular duo semigroup will be proved.

Theorem 1. *A semigroup S is a regular duo semigroup if and only if the conditions*

$$(L\cup LS)^2=L, \tag{1}$$

and

$$(R\cup SR)^2=R \tag{2}$$

hold for every left and every right ideal of S, respectively.

Proof. Let S be a regular duo semigroup. Then every one-sided ideal of S is two-sided, and

$$I\cap J=IJ \tag{3}$$

holds for any couple of ideals of S. (3) implies that every ideal I of S is globally idempotent, i.e.

$$I^2=I \tag{4}$$

for any ideal I of S. This implies both (1) and (2).

Conversely, suppose that S is a semigroup with properties (1) and (2) for every left and right ideal, respectively. Then (2) implies that each right ideal R of S is also a left ideal, and (1) implies that every left ideal L of S is two-sided. Therefore S is a duo semigroup. Finally (1) implies (4) for any ideal I of S, which is equivalent to the regularity of a duo semigroup. (See [5].)

Theorem 1 is completely proved.

Next utilizing the author's recent results concerning the (m,n)-ideals of regular duo semigroups (see [6]), one can prove the following result.

Theorem 2. *A semigroup S is a regular duo semigroup if and only if the relation*

$$(B\cup SB)^2=B=(B\cup BS)^2 \tag{5}$$

holds for any bi-ideal B of S.