87. On H-closedness and the Wallman H-closed Extensions. II*)

By Chien WENJEN
California State College at Long Beach, U. S. A.

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1. **Introduction.** By an extension of a space X is meant a space containing a dense set homeomorphic to X (also denoted by X). A point in the extension not belonging to X is represented by a family of closed sets in X with PFIP which consists of the intersections of X and the closures of the neighborhoods of the point. The collection of all maximal families of closed sets in X with PFIP and suitable topology then constitutes an H-closed extensions $\omega(X)$ of X, called the Wallman H-closed extensions and possesing properties similar to those of the Stone-Čech compactification $\beta(T)$ of a completely regular space T. In particular, continuous functions on X can be continuously extended over $\omega(X)$ and there is a variant of the Stone-Čech theorem [8, p. 153] for Hausdorff spaces.

There are two kinds of normal bases for spaces in literature: one is given by Fan and Gottesman for compactificating regular spaces [4] and the other is employed by Frink to identify complete regularity [6]. These bases are, in fact, equivalent in regular spaces. A new concept, called pseudo-normality which is similar to but more general than normality, is introduced as a characterization of complete regularity. The Fan-Gottesman compactification X^* of a completely regular space X is homeomorphic to the Stone-Čech compactification βX and is also homeomorphic to Aleksandrov $\alpha' X$ [1, p. 405].

The Stone-Weierstrass approximation theorem and the Tietze extension theorem will be generalized to Hausdorff spaces. Aleksandrov [2, Surveys, p. 54] and Pomonarov raised the question: for each completely regular space T whether the Stone-Weierstrass theorem holds in the Wallman H-closed extension $\omega(T)$ (topologically equivalent to $\tau(T)$ in [2]). A theorem due to Fan and Gottesman [4] sheds some light on the problem and an affirmative answer is given in § 4.

2. **The Wallman H-cosed extensions.**

Let X be a space, $\mathfrak{C}$ the family of all closed subsets of X, and $W(X)$ the collection of all subfamilies of $\mathfrak{C}$ which possess the PFIP and are

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