84. *The Additive Structure of the Unrestricted Z_p-Bordism Groups $\mathcal{O}_n(Z_p)$*

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Introduction. In this note we compute the additive structure of $\mathcal{O}_n(Z_p)$ and obtain that for $n \geqslant 0$,

$$\mathcal{O}_n(Z_p) \approx \begin{cases} \text{2-torsion} & \text{for } n \text{ odd,} \\ \text{free} + \text{2-torsion} & \text{for } n \text{ even,} \end{cases}$$

where the 2-torsion part consists of elements of order two.

We also compute the generators of $\mathcal{O}_n(Z_3)$ for $n \leqslant 7$, and study its connection with the Ω-module structure of $\mathcal{O}_*(Z_3)$ which we have determined in [5].

1. The additive structure of $\mathcal{O}_n(Z_p)$. We consider all (M^n, T) of Z_p-actions which form the Z_p-bordism group $\mathcal{O}_n(Z_p)$. First we shall need the exact sequence

$$0 \longrightarrow \Omega_n \xrightarrow{i_*} \mathcal{O}_n(Z_p) \xrightarrow{\nu} \mathfrak{M}_n(Z_p) \xrightarrow{\partial} \tilde{\Omega}_{n-1}(Z_p) \longrightarrow 0$$

which we already have in [5, Cororally 1.1]. Here $\tilde{\Omega}_{n-1}(Z_p)$ is the reduced, fixed point free, Z_p-bordism group, and $\mathfrak{M}_n(Z_p) = \sum_{k \geqslant 0} \Omega_{n-2k}(B(U(k_1) \times \cdots \times U(k_{p-1})))$, $k = k_1 + \cdots + k_{(p-1)/2}$. Moreover i_* is defined by $i_*[M^n] = [M \times Z_p, 1 \times \sigma] \in \mathcal{O}_n(Z_p)$ where σ is the map of period p which interchanges elements of Z_p; ν is defined by sending $[M^n, T] \in \mathcal{O}_n(Z_p)$ to the normal bundle over the fixed point set of T, $\sum_{k \geqslant 0} [\nu_k \to F_T^{n-2k}] \in \mathfrak{M}_n(Z_p)$, where $\nu_k \to F_T^{n-2k}$ is the complex k-dimensional normal bundle over the union F_T^{n-2k} of the $(n-2k)$-dimensional components of the fixed point set of T, and ∂ is defined by sending $\sum [V^{n-2k}, g_k] = \sum [\xi_k \to V^{n-2k}] \in \mathfrak{M}_n(Z_p)$ to the sphere bundles $\sum [S(\xi_k), \rho] \in \tilde{\Omega}_{n-1}(Z_p)$ where $\rho = \exp(2\pi i/p)$ and $\xi_k \to V^{n-2k}$ is the complex k-plane bundle classified by the map $g_k : V^{n-2k} \to B(U(k_1) \times \cdots \times U(k_{(p-1)/2}))$.

We also need several facts provided by Conner and Floyd in [3]:

For $X = B(U(k_1) \times \cdots \times U(k_{(p-1)/2}))$, $\Omega_n(X) \approx \sum_{j=0}^{n} H_j(X; \Omega_{n-j})$, [3, 15.2].

For a Ω-base $\{[S^{2i-1}, \rho]\}$ of $\tilde{\Omega}_*(Z_p)$, [3, 34.3], $[S^{2i-1}, \rho]$ has order p^{a+1} where $a(2p-2) < 2i-1 < (a+1)(2p-2)$, [3, 36.1].

And if $2i-1 = a(2p-2)+1$, then $p^a[S^{2i-1}, \rho] = b[S^1, \rho] \cdot [CP(p-1)]^a$ where $b \not\equiv 0 \pmod{p}$, [3, 36.2].

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