81. *Complex Powers of a System of Pseudo-differential Operators*

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§0. Introduction. In this paper we shall give complex powers of a system of pseudo-differential operators which is not necessarily elliptic. Complex powers of an elliptic pseudo-differential operator were defined by Seeley [5]. He constructed complex powers of a pseudo-differential operator $p(x, D_x)$ defined on a compact C^∞-manifold without boundary. Here we shall construct symbols for complex powers only by local calculation which works even for operators defined locally.

Recently Nagase-Shinkai [4] gave a concrete representation of complex powers of a pseudo-differential operator. They got the formula by using algebraic relation for the symbol of a pseudo-differential operator. But their method is not applicable to the case of systems, because they essentially used the commutativity of symbols.

We shall adopt the method of the Dunford integral for the symbol of the parametrix for $(p(x, D_x)-\zeta I)$. The relation between parametrices for $(p(x, D_x)-\zeta_1 I)$ and $(p(x, D_x)-\zeta_2 I)$, called the quasi-resolvent equation, plays an important role in place of the resolvent equation.

In the case of a single operator complex powers in the present paper coincide with those in [4]. We also note that complex powers of a parabolic system are asymptotically equal to operators with kernels whose supports lie in the half-space.

§1. Main theorem. Let $\lambda(\xi)$ be a fixed basic weight function, that is, a C^∞-function on R^n which have properties: $(1+|\xi|)^\rho \leqq \lambda(\xi) \leqq C_0(1+|\xi|)$ for some $\rho(0<\rho\leqq 1)$ and $|\partial_\xi^\alpha \lambda(\xi)| \leqq C_\alpha \lambda(\xi)^{1-|\alpha|}$ for any α (cf. [4]), where $\alpha=(\alpha_1, \cdots, \alpha_n)$ is a multi-index, $|\alpha|=\alpha_1+\cdots+\alpha_n$, and $\partial_\xi^\alpha = \partial_{\xi_1}^{\alpha_1}\cdots\partial_{\xi_n}^{\alpha_n}$. We denote by S_λ^m the set of all C^∞-symbols $p(x, \xi)$ on $R^n \times R^n$ satisfying, for any multi-index α, β, $|\partial_\xi^\alpha D_x^\beta p(x, \xi)| \leqq C_{\alpha,\beta}\lambda(\xi)^{m-|\alpha|}$ for some constant $C_{\alpha,\beta}$, and we define the pseudo-differential operator $p(x, D_x)$ of class S_λ^m by

$$p(x, D_x)u(x)=(2\pi)^{-n}\int e^{ix\cdot\xi}p(x, \xi)\hat{u}(\xi)d\xi,$$

where $D_x^\alpha=(-i\partial/\partial x_1)^{\alpha_1}\cdots(-i\partial/\partial x_n)^{\alpha_n}$ and $\hat{u}(\xi)=\mathcal{F}[u](\xi)$ denotes the Fourier transform of a rapidly decreasing function $u(x)$ defined by

$$\hat{u}(\xi)=\int e^{-ix\cdot\xi}u(x)dx.$$