80. *Stability, Attraction Properties and Asymptotic Equivalence of Dynamical Systems*

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1. Introduction. The concepts of stabilities and attraction properties such as the attractors and the region of attraction are rather important to determine the behaviors of the abstract dynamical systems defined on a metric space [1], [2].

In this paper we investigate the problem that to what extent the stability properties and the attraction properties are preserved through the asymptotic equivalence.

Main results obtained are Theorems 3.5, 3.6, 3.7, 3.8 and 3.10.

2. Standing notations. R is the real line. R^+ is the set of non-negative real numbers.

X is a metric space with its metric d.

$\pi_\alpha: X\times R\to X$ is a dynamical system defined on X.

$\pi_\alpha(p, \cdot)$ is the motion through the point p.

$L^+(p, \alpha)=\{x ; x$ is a positive limit point of $\pi_\alpha(p, \cdot)\}$.

$J^+(p, \alpha)=\{y \in X ; \exists\{x_n\}\subset X, \{t_n\}\subset R$, such that $x_n\to x$, $t_n\to+\infty$ and $\pi_\alpha(x_n, t_n)\to y\}$.

$L^+(p, \alpha)$ and $J^+(p, \alpha)$ are called respectively the positive limit set and the first positive prolongational limit set of a motion $\pi_\alpha(p, \cdot)$.

3. The preservation of stability and attraction properties through the asymptotic equivalence.

The asymptotic equivalence between two differential equations has been an interesting subject for many mathematicians [3]. The following Definition 3.1 is a generalization of this concept for the case of abstract dynamical systems.

Definition 3.1. *We say a dynamical system π_α is asymptotically equivalent to π_β on a subset S of X if*

$$(\forall p \in S, \forall q \in S) \quad d(\pi_\alpha(p, t), \pi_\beta(q, t))\to 0 \quad (t\to+\infty)$$

is valid, and denote this fact as follows:

$$\pi_\alpha \sim \pi_\beta \qquad (S).$$

The following proposition is trivial:

Proposition 3.2. *The asymptotic equivalence is symmetric as well as transitive.*

Corollary 3.2.1. *The asymptotic equivalence of the dynamical systems on a singleton is an equivalence relation.*