79. *On a Convergence Theorem for Sequences of Holomorphic Functions*

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(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1971)

Let D be the unit disk $\{|z|<1\}$ and C be its circumference $\{|z|=1\}$. For two numbers $\alpha, \beta, 0\leqq\alpha<\beta\leqq 2\pi$, we put

$$S(\alpha, \beta)=\text{the sector } \{z=re^{i\theta}\,;\ \alpha\leqq\theta\leqq\beta, 0\leqq r<1\},$$
$$C(\alpha, \beta)=\text{the arc } \{z=e^{i\theta}\,;\ \alpha\leqq\theta\leqq\beta\},$$
$$S_R(\alpha, \beta)=S(\alpha, \beta)\cap\{|z|<R\}, 0<R<1,$$
$$C_R(\alpha, \beta)=\text{the arc } \{z=Re^{i\theta}\,;\ \alpha\leqq\theta\leqq\beta\}.$$

We say that a function $f(z)$, holomorphic *on* $S(\alpha, \beta)$, *belongs to a class* $N_{(\alpha,\beta)}$ if

$$m(r, f\,;\ \alpha, \beta)=\int_\alpha^\beta \log^+|f(re^{i\theta})|d\theta \text{ is bounded for } 0\leqq r<1.$$

The class $N_{(0,2\pi)}$ is denoted simply by N and called the class of functions *of bounded characteristic* [1].

A function $f(z)$, holomorphic *in* $S(\alpha, \beta)$, is said to *belong to a class* $N^*_{(\alpha,\beta)}$ if $f(z)\in N_{(\alpha+\delta,\beta-\delta)}$ for every $\delta, 0<\delta<(\alpha+\beta)/2$.

It is proved in [2], as a localization of the Fatou's theorem, that

A function $f(z)$, holomorphic in $S(\alpha, \beta)$, can be written as a quotient of two bounded functions in $S(\alpha+\delta, \beta-\delta)$ for every $\delta, 0<\delta<(\alpha+\beta)/2$, if and only if $f(z)$ belongs to $N^*_{(\alpha,\beta)}$. In particular, a function $f(z)$ of the class $N^*_{(\alpha,\beta)}$ has finite angular limits almost everywhere on $C(\alpha, \beta)$, and if $\{z_n\}$ are the zeros of $f(z)$ in $S(\alpha+\delta, \beta-\delta)(\delta>0$ is fixed), we have

$$\Sigma(1-|z_n|)<\infty.$$

In this note we will prove, using the method of [2], a localization of the theorem of Khintchine-Ostrovski [3, p. 83], i.e.,

Theorem 1. *Let a sequence $\{f_n(z)\}\subset N^*_{(\alpha,\beta)}$ satisfy the conditions*:

(i)

$$\int_\alpha^\beta \log^+|f_n(re^{i\theta})|d\theta\leqq K, 0\leqq r<1, \tag{1}$$

where K is a constant independent of n and r.

(ii) *There is a set $E\subset C(\alpha, \beta)$, meas$(E)>0$, on which $\{f_n(e^{i\theta})\}$ converges in measure, where $f_n(e^{i\theta})$ denotes the radial limit of $f_n(z)$ at $e^{i\theta}$.*

Then $\{f_n(z)\}$ converges to a function $f(z)$ uniformly on any compact set in $S(\alpha, \beta)$. $f(z)$ is holomorphic in $S(\alpha, \beta)$ and has finite radial limit $f(e^{i\theta})$ at almost every point $e^{i\theta}\in C(\alpha, \beta)$, and $\{f_n(e^{i\theta})\}$ converges in measure to $f(e^{i\theta})$ on the set E.