76. *The Theory of Nuclear Spaces Treated by the Method of Ranked Space.* I

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1. Introduction. In this paper we will show that the nuclear space in Gel'fand [2] can be considered as the limiting space of finite dimensional Euclidean space, when the limiting process is taken in the sense of ranked space given by K. Kunugi.

Following Gel'fand [2], the nuclear space Φ is a countably Hilbert space $\Phi=\bigcap_{i=1}^{\infty}\Phi_i$, in which for any m there is an n such that the mapping T_m^n, $m<n$, of the space Φ_n into the space Φ_m is nuclear, i.e., has the form

$$T_m^n\varphi=\sum_{k=1}^{\infty}\lambda_k(\varphi,\varphi_k)_n\psi_k,\qquad \varphi\in\Phi_n,$$

where $\{\varphi_k\}$ and $\{\psi_k\}$ are orthonormal systems of vectors in the space Φ_n and Φ_m respectively, $\lambda_k>0$ and $\sum_{k=1}^{\infty}\lambda_k$ converges.

§ 2. Definition of neighbourhoods. Let the mappings $T_{n_0}^{n_1}, T_{n_1}^{n_2}, \cdots, T_{n_{i-1}}^{n_i}, T_{n_i}^{n_{i+1}}, \cdots, (n_0=1<n_1<n_2<\cdots<n_{i-1}<n_i<n_{i+1}<\cdots)$ be nuclear operators in the nuclear space Φ. As shown in § 1, we can write $T_{n_i}^{n_{i+1}}(i=0,1,2,\cdots)$ in the following form

$$T_{n_i}^{n_{i+1}}\varphi=\sum_{k=1}^{\infty}\lambda_{k,n_i,n_{i+1}}(\varphi,\varphi_{k,n_{i+1}})_{n_{i+1}}\varphi_{k,n_i}$$

where $\lambda_{k,n_i,n_{i+1}}>0$ and $\sum_{k=1}^{\infty}\lambda_{k,n_i,n_{i+1}}<\infty$. Now, we define

$$U_i(0,\varepsilon,m)=\left\{T_{n_{i-1}}^{n_i}\varphi:\varphi\in\Phi_{n_i}\cap\Phi\,\middle|\,\left\|\sum_{k=1}^{m}\lambda_{k,n_{i-1},n_i}(\varphi,\varphi_{k,n_i})\varphi_{k,n_{i-1}}\right\|_{n_{i-1}}<\varepsilon\right\}$$

as neighbourhoods of the origin of Φ and we call them neighbourhoods of rank i.

Lemma 1. *If we have* $m_i\leqq m_{i+1}$ and $(\sum_{k=1}^{\infty}\lambda_{k,n_{i-1},n_i})\varepsilon_{i+1}\leqq\varepsilon_i$, *we obtain*

$$U_i(0,\varepsilon_i,m_i)\supseteq U_{i+1}(0,\varepsilon_{i+1},m_{i+1}).$$

Proof. Suppose that $U_{i+1}(0,\varepsilon_{i+1},m_{i+1})\ni T_{n_i}^{n_{i+1}}\varphi$, $\varphi\in\Phi_{n_{i+1}}\cap\Phi$, then $\|\sum_{k=1}^{m_{i+1}}\lambda_{k,n_i,n_{i+1}}(\varphi,\varphi_{k,n_{i+1}})_{n_{i+1}}\varphi_{k,n_i}\|_{n_i}<\varepsilon_{i+1}$. Hence we obtain

$$\begin{aligned}&\left\|\sum_{k=1}^{m_i}\lambda_{k,n_{i-1},n_i}(T_{n_i}^{n_{i+1}}\varphi,\varphi_{k,n_i})_{n_i}\varphi_{k,n_{i-1}}\right\|_{n_{i-1}}\\&=\left\|\sum_{k=1}^{m_i}\lambda_{k,n_{i-1},n_i}\left(\sum_{h=1}^{\infty}\lambda_{h,n_i,n_{i+1}}(\varphi,\varphi_{h,n_{i+1}})_{n_{i+1}}\varphi_{h,n_i},\varphi_{k,n_i}\right)_{n_i}\varphi_{k,n_{i-i}}\right\|_{n_{i-1}}\\&\leqq\left(\sum_{k=1}^{m_i}\lambda_{k,n_{i-1},n_i}\right)\left\|\sum_{h=1}^{m_{i+1}}\lambda_{h,n_i,n_{i+1}}(\varphi,\varphi_{h,n_{i+1}})_{n_{i+1}}\varphi_{h,n_i}\right\|_{n_i}\\&<\left(\sum_{k=1}^{\infty}\lambda_{k,n_{i-1},n_i}\right)\varepsilon_{i+1}\leqq\varepsilon_i,\text{ then }T_{n_{i-1}}^{n_i}(T_{n_i}^{n_{i+1}}\varphi)\in U_i(0,\varepsilon_i,m_i).\end{aligned}$$