110. *An Analogue of the Paley-Wiener Theorem for the Heisenberg Group*

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1. Introduction. Let $\boldsymbol{R}$ (resp $\boldsymbol{C}$) be the real (resp. complex) number field as usual. Let G be the n-th Heisenberg group, i.e. the group of all real matrices of the form

$$\begin{pmatrix} 1 & a & c \\ 0 & I_n & b \\ 0 & 0 & 1 \end{pmatrix} \tag{1.1}$$

where $a=(a_1,\cdots,a_n)\in \boldsymbol{R}^n$, $b={}^t(b_1,\cdots,b_n)\in \boldsymbol{R}^n$, $c\in \boldsymbol{R}$ and I_n is the identity matrix of n-th order. Let H be the abelian normal subgroup consisting of the elements of the form (1.1) with $a=0$. For any real η we denote by χ_η the unitary character of H defined by $\chi_\eta: \begin{pmatrix} 1 & 0 & c \\ 0 & I_n & b \\ 0 & 0 & 1 \end{pmatrix} \to e^{2\pi i\eta c}$. Let U^η be the unitary representation of G induced by χ_η. Then the Plancherel theorem can be proved by means of $U^\eta(\eta\in \boldsymbol{R})$ (see e.g. [4]). However, as we have seen in the case of euclidean motion group ([2]), in order to prove an analogue of the Paley-Wiener theorem we have to consider the representations which have more parameters.

Let $\hat{H}$ be the dual group of H. In this paper we consider the Fourier transform defined on $\hat{H}\cong \boldsymbol{R}^{n+1}$.

Let $C_c^\infty(G)$ be the set of all infinitely differentiable functions on G with compact support. For any $\xi\in \boldsymbol{R}^n$ and $\eta\in \boldsymbol{R}$ we denote by $U^{\xi,\eta}$ the unitary representation of G induced by the unitary character $\chi_{\xi,\eta}$ of H: $\chi_{\xi,\eta}\begin{pmatrix} 1 & 0 & c \\ 0 & I_n & b \\ 0 & 0 & 1 \end{pmatrix}=e^{2\pi i\langle \xi,b\rangle+2\pi i\eta c}$. We define the (operator valued) Fourier transform T_f of $f\in C_c^\infty(G)$ by

$$T_f(\xi,\eta)=\int_G f(g)U_g^{\xi,\eta}dg,$$

where dg is the Haar measure on G. Then $T_f(\xi,\eta)$ is an integral operator on $L_2(\boldsymbol{R}^n)$ (§2). Denote by $K_f(\xi,\eta;x,y)$ $(x,y\in \boldsymbol{R}^n)$ be the kernel function of $T_f(\xi,\eta)$. We shall call K_f the scalar Fourier transform of f.

The purpose of this paper is to determine the image of $C_c^\infty(G)$ by the scalar Fourier transform (analogue of the Paley-Wiener theorem).