109. *An Analogue of the Paley-Wiener Theorem for the Euclidean Motion Group*

By Keisaku KUMAHARA*) and Kiyosato OKAMOTO**)

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1. Introduction. The purpose of this paper is to prove an analogue of the Paley-Wiener theorem for the group G of the motions of the n-dimensional euclidean space.

Let $\hat{G}$ be the set of all equivalence classes of irreducible unitary representations of G. Let $L_2(G)$ (resp. $L_2(\hat{G})$) be the Hilbert space of all square integrable functions on G (resp. $\hat{G}$) with respect to the Haar measure (resp. the Plancherel measure). Then the Plancherel theorem states that the Fourier transform gives an isometry of $L_2(G)$ onto $L_2(\hat{G})$ (see § 2).

Let $C_c^\infty(G)$ be the space of all infinitely differentiable functions with compact support on G. By an analogue of the Paley-Wiener theorem we mean the characterization of the image of $C_c^\infty(G)$ by the Fourier transform.

As a number of articles ([1], [2], [4], [7]–[9] and etc.) indicate, in order to attack the problem one has to consider the Fourier-Laplace transforms of $C_c^\infty(G)$ which are (operator-valued) entire analytic functions "of exponential type" on a certain complex manifold. In general, $\hat{G}$ is not a C^∞ manifold but the space of all orbits in a real analytic manifold by actions of the "Weyl group" which gives equivalence relations. The Fourier-Laplace transform T_f of an element f of $C_c^\infty(G)$ is defined on the "complexification" of this real analytic manifold and satisfies certain functional equations derived from the actions of the Weyl group.

Detailed proofs will appear elsewhere.

2. Preliminaries. Let G be the group of motions of n-dimensional euclidean space $\boldsymbol{R}^n$. Then G is realized as the group of $(n+1)\times(n+1)$-matrices of the form $\begin{pmatrix} k & x \\ 0 & 1 \end{pmatrix}$, $(k \in SO(n), x \in \boldsymbol{R}^n)$. Let K and H be the closed subgroups of the elements $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, $(k \in SO(n))$ and $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$, $(x \in \boldsymbol{R}^n)$, respectively. Then H is an abelian normal subgroup of G and G is the semidirect of H and K. We normalize the Harr measure dg on G such that $dg=dxdk$, where $dx=(2\pi)^{-n/2}dx_1\cdots dx_n$

*) Department of Applied Mathematics, Osaka University.
**) Department of Mathematics, Hiroshima University.