# 108. *On the Asymptotic Behaviors of Solutions of Difference Equations. II*

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As for the applications of Lyapunov functions to the stability problems of difference equations with discrete variable, we can find some results in [2, 3, 5], and [4] concerning the criteria of Popov type for the absolute stability. In this paper, we shall show some other results including the construction of Lyapunov functions, that is, the so-called converse theorems, and the applications to perturbed systems.

The following is a result to show the existence of Lyapunov functions for linear systems, which will be often used to discuss the stability problems for perturbed systems.

**Theorem 1.** *Suppose that $A(t)$ be an $n\times n$ matrix defined for $t\in I_\infty$, and the trivial solution of*

(1) $$x(t+1)=A(t)x(t), \quad x(t_0)=x_0, \quad t\geqq t_0$$

*is generalized exponentially asymptotically stable, where $I_\infty$ is a set of nonnegative integers and $t_0\in I_\infty$. Then there exists a function $V(t,x)$ satisfying the following conditions:*

(a) *$V(t,x)$ is defined for $t\in I_\infty$ and $|x|<\infty$, Lipschitzian in $x$ for a function $K(t)$;*

(b) $$|x|\leqq V(t,x)\leqq K(t)|x|, \quad t\in I_\infty, \quad |x|<\infty;$$

(c) *for any solution $x(t)$ of* (1),

$$\Delta V(t,x(t))\leqq -(1-\exp(-\Delta p(t)))V(t,x(t)), \qquad t\geqq t_0.$$

This theorem will be proved by an analogous method as in differential equations, if we define a function $V(t,x)$ such that

$$V(t,x)=\sup_{\sigma\in I_\infty}|x(t+\sigma,t,x)|\,e^{p(t+\sigma)-p(t)}.$$

For the definition of the generalized exponentially asymptotic stability, see [1].

**Theorem 2.** *Suppose that*

(i) *$A(t)$ is defined for $t\in I_\infty$, and the trivial solution of* (1) *is generalized exponentially asymptotically stable;*

(ii) *$F(t,x)$ is defined for $t\in I_\infty$ and $|x|<\rho$, and $|F(t,x)|\leqq g(t,|x|)$, $t\in I_\infty$, $|x|<\rho$, where $g(t,r)$ is defined for $t\in I_\infty$ and $0\leqq r<\infty$, $g(t,0)\equiv 0$, and nondecreasing in $r$ for any $t$.*

*Then the stability or asymptotic stability of the trivial solution of*