107. *On the Asymptotic Behaviors of Solutions of Difference Equations. I*

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Introduction. In a view point of engineering, difference equations are often used to analyze the so-called sample-data systems, in which the stability problems are considered to be very important. It seems, however, that, if we are concerned with the problems of the asymptotic behaviors for difference equations, not so many papers have been appeared so far.

The purpose of this paper is to state some results on certain types of the asymptotic behaviors of solutions of difference equations with discrete variable, but it is noted that we can obtain various results concerning the other problems, for example, the problems of boundedness, total stability, integral stability, almost periodicity, and so on, and is also expected that some results shown in this paper can be applied to the error estimation in the numerical analysis.

0. Preliminaries. We first summarize some lemmas which are often used to prove the results in the following sections. The other types of comparison theorems will be refered to [2]. We denote by I_m a set of discrete points t_0+k $(k=0, 1, \cdots, m; 0<m\leqq\infty)$, and the norms of matrices are defined suitably.

Lemma 1. *Let $g(t, r)$ be defined for $t\in I_{N-1}$ and $0\leqq r<\infty$, and nondecreasing in r for any fixed t. Then, if the function $u(t)$ and $r(t)(t\in I_N)$ satisfy the relations*

$$u(t)\leqq u_0+\sum_{s=t_0}^{t-1} g(s, u(s)), \qquad r(t)=r_0+\sum_{s=t_0}^{t-1} g(s, r(s))$$

respectively, where u_0 and r_0 are constant, there holds an inequality $u(t)\leqq r(t)$, $t\in I_N$, provided $u_0\leqq r_0$.

It is easily observed that the same result is valid, if we replace the inequality and equation by the followings respectively:

$$\Delta u(t)\leqq g(t, u(t)), \qquad \Delta r(t)=g(t, r(t)),$$

where Δ is an operator such that $\Delta\varphi(t)=\varphi(t+1)-\varphi(t)$. Such a replacement may also be applicable to Lemma 3.

Lemma 2. *Let $f(t, x)$ be defined for $t\in I_N$ and $|x|<\infty$, and $g(t, r)$ satisfy the same condition as above. Then, if an inequality $|f(t, x)|\leqq g(t, |x|)$ is satisfied, there holds an inequality $|x(t)|\leqq r(t)$, $t\in I_{N+1}$,*