103. *On Some Examples of Non-normal Operators*

By Masatoshi FUJII
Fuse Senior Highschool

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1971)

1. Introduction. Following after the terminology of Halmos [4], consider a (bounded linear) operator T acting on a Hilbert space $\mathfrak{H}$. As usual, we shall call

$$W(T)=\{(Tx|x)\,;\,\|x\|=1\}$$

the *numerical range* of T and

$$r(T)=\sup\{|\lambda|\,;\,\lambda\in\sigma(T)\}$$

the *spectral radius* of T, where $\sigma(T)$ is the spectrum of T. An operator T is called *normaloid* if $\|T\|=r(T)$ and *convexoid* if $\bar{W}(T)=\mathrm{co}\,\sigma(T)$ where $\bar{W}(T)$ is the closure of $W(T)$ and $\mathrm{co}\,S$ is the convex hull of S. We shall also say that an operator T satisfies the *growth condition* (G_1) if

$$\|(T-\lambda)^{-1}\|\leqq\frac{1}{\mathrm{dist}\,(\lambda,\sigma(T))}$$

for any $\lambda\notin\sigma(T)$. An operator satisfying the condition (G_1) is a convexoid.

In a recent paper [7], Luecke proves the following theorem which gives a method of construction of operators satisfying the condition (G_1):

Theorem A (Luecke). *If A is an operator acting on a Hilbert space $\mathfrak{H}$, then there is an operator B acting on a Hilbert space $\mathfrak{K}$ such that their direct sum $T=A\oplus B$ acting on $\mathfrak{H}\oplus\mathfrak{K}$ satisfies the condition* (G_1).

In his proof, the desired B satisfies the normality and $\bar{W}(A)=\sigma(B)$. Using Theorem A, he can prove that there is an operator satisfying the condition (G_1) which is not a normaloid.

Inspired by Luecke's work and a seminar talk of R. Nakamoto (Theorem 5 in the below), we shall adapt the method to construct another examples of operators in § 2 and apply them to study for a few relations between classes of non-normal operators in § 3.

For our purpose, we shall introduce two classes of operators which are systematically discussed by Hildebrandt [5] without their names:

Definition B. An operator T is called a *numeroid* (resp. *spectroid*) if the closed numerical range $\bar{W}(T)$ (resp. the spectrum $\sigma(T)$) is a spectral set for T in the sense of von Neumann [8].