101. *A Remark on Perturbation of m-accretive Operators in Banach Space*

By Yoshio KONISHI
Department of Mathematics, University of Tokyo

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1. Introduction. Let X be a real Banach space with the norm denoted by $\|\cdot\|$. By definition a (possibly) multiple-valued operator A in X is *accretive* if for each $\lambda>0$ and $u, v \in D(A)$,

$$\|x-y\| \geq \|u-v\| \quad \text{whenever} \quad x \in (I+\lambda A)u, y \in (I+\lambda A)v.$$

An accretive operator A in X is said to be *m-accretive* if $R(I+A)=X$. For the notion of "multiple-valued" operator, we refer to T. Kato [6], § 2.

The purpose of the present paper is to give a criterion for the m-accretiveness of the sum of two m-accretive operators in X and then apply it to a certain nonlinear partial differential equation. Our result may be considered to constitute an analogue of the result of H. Brezis, M. G. Crandall and A. Pazy [1] for perturbation of maximal monotone sets.

2. A perturbation lemma. Let A and B be m-accretive operators in X. As usual we define the *Yosida approximation* $B_\varepsilon (\varepsilon>0)$ of B by

$$B_\varepsilon = \varepsilon^{-1}\{I-(I+\varepsilon B)^{-1}\},$$

which is a single-valued Lipschitz continuous operator defined on all of X. It is easily seen that $A+B_\varepsilon$ is again m-accretive and accordingly that for each $f \in X$ there exists a unique solution $u_\varepsilon \in D(A)$ of the equation

$$u_\varepsilon + y_\varepsilon + B_\varepsilon u_\varepsilon = f, \qquad y_\varepsilon \in Au_\varepsilon. \tag{2.1}$$

Lemma 1. *Assume that X is a real Banach space with the uniformly convex dual space X^* and that A and B are m-accretive operators in X such that $D(A) \cap D(B) \ni 0$. If for each fixed $f \in X$ $\|B_\varepsilon u_\varepsilon\|$ in* (2.1) *is bounded as ε tends to zero, then $A+B$ is m-accretive.*

We notice that if X^* is uniformly convex, the duality map F defined as

$$Fu = \{u^* \in X^* ; (u, u^*) = \|u\|^2 = \|u^*\|^2\}, \qquad u \in X,$$

is single-valued and is uniformly continuous on any bounded set (T. Kato [5]).

Proof of Lemma 1. The argument of the proof is standard (see Y. Kōmura [7] and T. Kato [5,6]). Since $D(A+B) \ni 0$, there is no loss