137. *Determination of $\tilde{K}_O(X)$ by $\tilde{K}_{SO}(X)$ for 4-Dimensional CW-Complexes*

By Yutaka ANDO
Mathematical Institute, Tokyo University of Fisheries

(Comm. by Kenjiro SHODA, M. J. A., Sept. 13, 1971)

0. For a connected finite 4-dimensional CW-complex X we denote the group of stable vector bundles over X by $\tilde{K}_O(X)$, and the group of orientable stable vector bundles over X by $\tilde{K}_{SO}(X)$. In the previous paper [2] S. Sasao and the author determined the group structures of $\tilde{K}_{SO}(X)$ by cohomology rings. In this note we shall determine the relation between $\tilde{K}_O(X)$ and $\tilde{K}_{SO}(X)$. Our results include that $\tilde{K}_O(X) \cong \tilde{K}_{SO}(X) + H^1(X; Z_2)$ if and only if $Sq^1H^1(X; Z_2)=0$. The author wishes to thank Professor S. Sasao for his valuable suggestions.

1. We can easily prove the following

Proposition 1. *The sequence*

$$0 \longrightarrow \tilde{K}_{SO}(X) \xrightarrow{i} \tilde{K}_O(X) \xrightarrow{W_1} H^1(X; Z_2) \longrightarrow 0$$

is exact, where i is a map which forgets the orientation and W_1 maps each class $[\xi]$ to the first Whitney class $W_1(\xi)$ of a bundle ξ which represents $[\xi]$.

This proposition shows that $\tilde{K}_O(X)$ is an element of $EXT(H^1(X; Z_2), \tilde{K}_{SO}(X))$. So we investigate this group.

Proposition 2. *There exists an isomorphism*

$$\varphi: EXT(H^1(X; Z_2), \tilde{K}_{SO}(X) \longrightarrow \sum_{i=1}^{r} (\tilde{K}_{SO}(X)/2\tilde{K}_{SO}(X))_i$$

where $r=\dim H^1(X; Z_2)$.

Proof. We assume that $H^1(X; Z_2) \cong \sum_{i=1}^{r} Z_2[\alpha_i]$, where [ ] denotes the generator. Consider the follwing exact sequence

$$0 \longrightarrow H \xrightarrow{i} F \xrightarrow{j} H^1(X; Z_2) \longrightarrow 0$$

where F is a free abelian group generated by $\{f_i\}$ such that $j(f_i)=\alpha_i$. By $\{h_i\}$ we denote generators of H corresponding to $\{2f_i\}$ via i. Then we know that there exists an isomorphism

$\rho: EXT(H^1(X; Z_2), \tilde{K}_{SO}(X)) \rightarrow HOM(H, \tilde{K}_{SO}(X))/\text{image } HOM(F, \tilde{K}_{SO}(X))$ defined as follows. For an exact sequence

$$0 \longrightarrow \tilde{K}_{SO}(X) \longrightarrow G \longrightarrow H^1(X; Z_2) \longrightarrow 0 ,$$

we take a set $\{g_i\}$ of elements of G going to $\{\alpha_i\}$. And we take a set $\{\gamma_i\}$ of elements of $\tilde{K}_{SO}(X)$ going to $\{2g_i\}$. Now we put $\rho(G)(h_i)=\gamma_i$ then $\rho(G)$ is uniquely defined as an element of $HOM(H, \tilde{K}_{SO}(X))/2HOM(H, \tilde{K}_{SO}(X)) \cong HOM(H, \tilde{K}_{SO}(X))/\text{image } HOM(F, \tilde{K}_{SO}(X))$. Let $p: \tilde{K}_{SO}(X) \rightarrow \tilde{K}_{SO}(X)$