# 65. *Theorems on the Finite-dimensionality of Cohomology Groups. II*

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The purpose of this note is to present some theorems on the finite-dimensionality of cohomology groups attached to a system of linear differential equations, which supplement our preceding note [4]. The method of the proof is almost the same as that employed in the above quoted note, that is, it essentially relies on the local analysis of the solution sheaf of the system under consideration and the comparison of the topological structures which can be naturally introduced to the cohomology group under consideration in two ways. The local analysis of the solution sheaf may be used to prove that the two topologies coincide in the case which we treat in this note. Note that Professor Guillemin has announced results close to ours in a recent paper [2]. It seems that he has used the so-called sub-elliptic estimates, while our method essentially relies on Sato's theory of micro-functions. The employment of hyperfunctions, which allows us to employ linear (pseudo-)differential operators *of infinite order*, makes our results applicable to very general situations (, though we restrict ourselves to the real analytic category, that is, the manifold under consideration is real analytic and the coefficients of the differential operators are real analytic). As for the theory of micro-functions we refer to Sato [8] and Sato, Kawai and Kashiwara [9]. We also refer to Sato, Kawai and Kashiwara [9] for the theory of pseudo-differential operators of infinite order and the micro-local analysis of systems of linear (pseudo-) differential equations.

We use the same notations as in our previous note [4] and do not repeat their definitions. The details of this note will appear somewhere else with some technical improvements of the results.

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**Theorem 1.** *Let $\mathfrak{M}$ be an admissible system of linear differential equations defined on a compact manifold $M$.*

*Case* (i) *Assume that the generalized Levi form attached to $V$ has $q$ negative eigenvalues on $V_R = V \cap S^*M$. Then*

$$\dim_C \mathrm{Ext}^i(M, \mathfrak{M}, \mathcal{A}) = \dim_C \mathrm{Ext}^i(M, \mathfrak{M}, \mathcal{B}) < \infty \qquad \text{for } i < q.$$