91. *On Hypersurfaces which are Close to Spheres*

By Kanji MOTOMIYA
Nagoya Institute of Technology

(Comm. by Kinjirô KUNUGI, M. J. A., June 2, 1972)

0. Some characterizations of the sphere among the closed strictly convex hypersurfaces in R^{n+1} were given in [1].
In particular, the following theorem holds:
A closed strictly convex hypersurface with $K_{n-1}/K_n = r$ is a hypersphere of radius r, where K_{n-1} is the $(n-1)$-th mean curvature and K_n is the Gaussian curvature.
Then, we prove

Theorem. *Let M be a closed strictly convex hypersurface in $R^{n+1}(n \geqslant 2)$. If the function K_{n-1}/K_n on M is sufficiently close to r, then M is arbitrary close to a hypersphere of radius r in the sense that it can be enclosed between two concentric hyperspheres whose radius is arbitrarily close to r.*
For the case where $n=2$, D. Koutroufiotis proved in [3]. Our proof of theorem is the same method of his proof in [3].

1. For the sake of simplicity, we shall assume our manifolds and mappings to be of class C^∞.
Let R^{n+1} be the $(n+1)$-dimensional euclidean space.
By a hypersurface in R^{n+1} we mean a n-dimensional connected manifold M with an immersion x.
Suppose M to be oriented. Then to $p \in M$, there is a uniquely determined unit normal vector $\xi(p)$ at $x(p)$.
We put

$$\mathrm{I} = dx \cdot dx, \qquad \mathrm{II} = -d\xi \cdot dx.$$

Let $k_1, \cdots, k_n$, are called the principal curvatures, be the eigenvalues of II relative to I. The i-th mean curvature K_i $(1 \leqslant i \leqslant n)$ is given by the i-th elementary symmetric function divided by $\binom{n}{i} = n!/i!(n-i)!$ i.e.,

$$\binom{n}{i} K_i = \sum k_1 \cdots k_i.$$

In particular, $K_n = k_1 \cdots k_n$ is called the Gaussian curvature. We shall consider closed strictly convex hypersurfaces i.e., compact hypersurfaces for which the Gaussian curvature K_n never vanishes on M.

We shall assume that the normal vector ξ is interior. Let S^n be the unit sphere in R^{n+1}. We denote by g the induced Riemannian metric on S^n.