90. The Theory of Nuclear Spaces Treated by the Method of Ranked Space. VII

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In this paper we study a measure in the extended nuclear space, which is investigated in the papers [3]–[7].

§9. Measure. The nuclear space Φ following Gel'fand is constructed in a countably Hilbert space $\Phi=\bigcap_{i=1}^{\infty}\Phi_i$. From now on we shall write $\{\varphi_k\}_{n=1,2,\dots}$ in place of $\{\varphi_{k,n_1}\}_{k=1,2,\dots}$, which is an orthomormal system in the Hilbert space Φ_{n_1}.

Definition 14. Let A be a Borel set in n-dimensional space E_n generated by finite set $\{\varphi_k\}_{k=1,\dots,n}$. And we define a set Z such that

$$Z=\left\{\varphi\in\hat{\Phi},\ \sum_{i=1}^{n}(\varphi,\varphi_i)\varphi_i\in A\right\}.$$

We call it a Borel cylinder set Z with Borel base A in subspace E_n.

Thus the cylinder sets form an algebra of sets, that is,

(1) The complement of any Borel cylinder set is a Borel cylinder set.

(2) The intersection of any two Borel cylinder sets is a Borel cylinder set.

(3) The union of any two Borel cylinder sets is a Borel cylinder set.

Now, we shall extend the class of the Borel cylinder sets.

Let $\mathfrak{K}_i$ be the class of the Borel cylinder sets with Borel base in E_i.

Next, let $\mathfrak{B}_0$ be all countable unions of the elements in $\bigcup_{i=1}^{\infty}\mathfrak{K}_i$ and all complements of such unions. And we call $\mathfrak{B}_0$ Borel sets of the zeroth class. Suppose that Borel sets of class β have already been defind, where β is any ordinal number less than α such that $\alpha<\Omega$.

Then let $\mathfrak{B}_\alpha$ be all countable unions of the elements of class less than α and all complements of such unions.

Thus $\mathfrak{B}_\alpha$ is defined for all transfinite ordinal numbers less than Ω. And we call "the element of $\bigcup_{\alpha<\Omega}\mathfrak{B}_\alpha$" Borel set of Borel cylinder set.

Now, we shall define a Gaussian measure for the Borel cylinder set.

Definition 15. For the Borel cylinder set Z with Borel base A in subspace E_n, we define $\mu(Z)$ such that

$$\mu(Z)=\frac{1}{(2\pi)^{n/2}}\int_A \exp\left(\frac{-1}{2}\left[\sum_{i=1}^{n}|(\varphi,\varphi_i)|^2\right]\right)d\varphi,$$

where $d\varphi$ is Lebesgue measure with respect to the scalar product

$$(\varphi,\varphi)=\sum_{i=1}^{n}|(\varphi,\varphi_i)|^2 \qquad \text{in } E_n.$$