89. On Normal Approximate Spectrum. III

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1. **Introduction.** In the previous notes [3] and [5], we have discussed certain properties of the normal approximate spectra of operators on a Hilbert space $\mathfrak{H}$. A complex number λ is an *approximate propervalue* of T acting on $\mathfrak{H}$ if there is a sequence $\{x_n\}$ of unit vectors such that

$$(*) \qquad \|(T-\lambda)x_n\| \to 0 \qquad (n\to\infty).$$

The set $\pi(T)$ of all approximate propervalues is called the *approximate spectrum* of T. If there exists a sequence $\{x_n\}$ of unit vectors satisfying $(*)$ and

$$(**) \qquad \|(T-\lambda)^* x_n\| \to 0 \qquad (n\to\infty),$$

then λ is called a *normal approximate propervalue* of T, which is occasionally discussed by Hildebrandt [7], Stampfli [11] and Yoshino [12]. The set $\pi_n(T)$ of all normal approximate propervalues of T is called the *normal approximate spectrum* of T. In general, $\pi_n(T)$ is a compact set in the plane and possibly void. Several equivalent conditions are discussed in [3], [5] and [9].

In the present note, we shall discuss some additional properties of the normal approximate spectra of operators. In § 2, we shall give a characterization of convexoids in terms of the normal approximate spectra. In a certain sense, a convexoid has sufficiently many normal approximate propervalues (Theorem 1), which is suggested by Prof. Z. Takeda, to whom the authors express their heaty thanks. In § 3, the normal approximate spectrum of the tensor product of operators is observed.

2. **A characterization of convexoids.** An operator T acting on a Hilbert space $\mathfrak{H}$ is called a *convexoid* if

$$(1) \qquad \bar{W}(T) = \mathrm{co}\, \sigma(T),$$

where $\bar{W}(T)$ is the closure of the numerical range $W(T)$ given by

$$(2) \qquad W(T) = \{(Tx \mid x)\,;\, \|x\| = 1\},$$

co S is the convex hull of S, and $\sigma(T)$ is the spectrum of T. The following theorem is suggested by Prof. Z. Takeda:

Theorem 1. *An operator T is a convexoid if and only if the closed numerical range $\bar{W}(T)$ is spanned by the normal approximate*

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